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Measuring Hidden Constructs
Introduction to Latent Factor Analysis

Brandon L. Bretl, PhD
Assistant Professor
School of Education
College of Education & Psychology
University of Texas at Tyler

Outline

- Latent Constructs
- Terminology, CFA and EFA
- Example 1: moral foundations theory
- Example 2: geology
- Some Practical Considerations
- Tools

Latent Constructs

Latent Constructs

Latent Factor Analysis

PATIENT HEALTH QUESTIONNAIRE (PHQ-9)

NAME:_____ **DATE:**_____

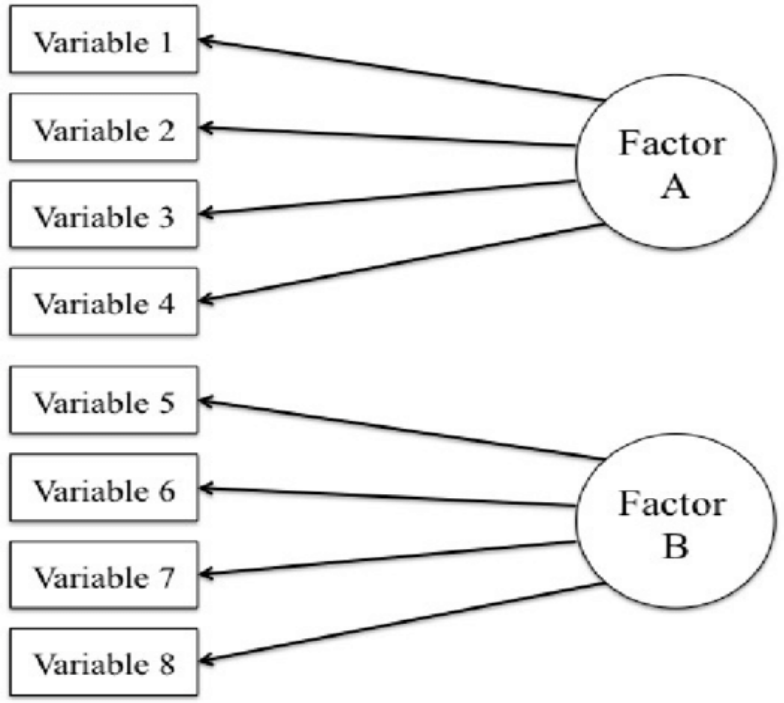
Over the last *2 weeks*, how often have you been bothered by any of the following problems?
(use "✓" to indicate your answer)

	Not at all	Several days	More than half the days	Nearly every day
1. Little interest or pleasure in doing things	0	1	2	3
2. Feeling down, depressed, or hopeless	0	1	2	3
3. Trouble falling or staying asleep, or sleeping too much	0	1	2	3
4. Feeling tired or having little energy	0	1	2	3
5. Poor appetite or overeating	0	1	2	3

Latent Factor Analysis

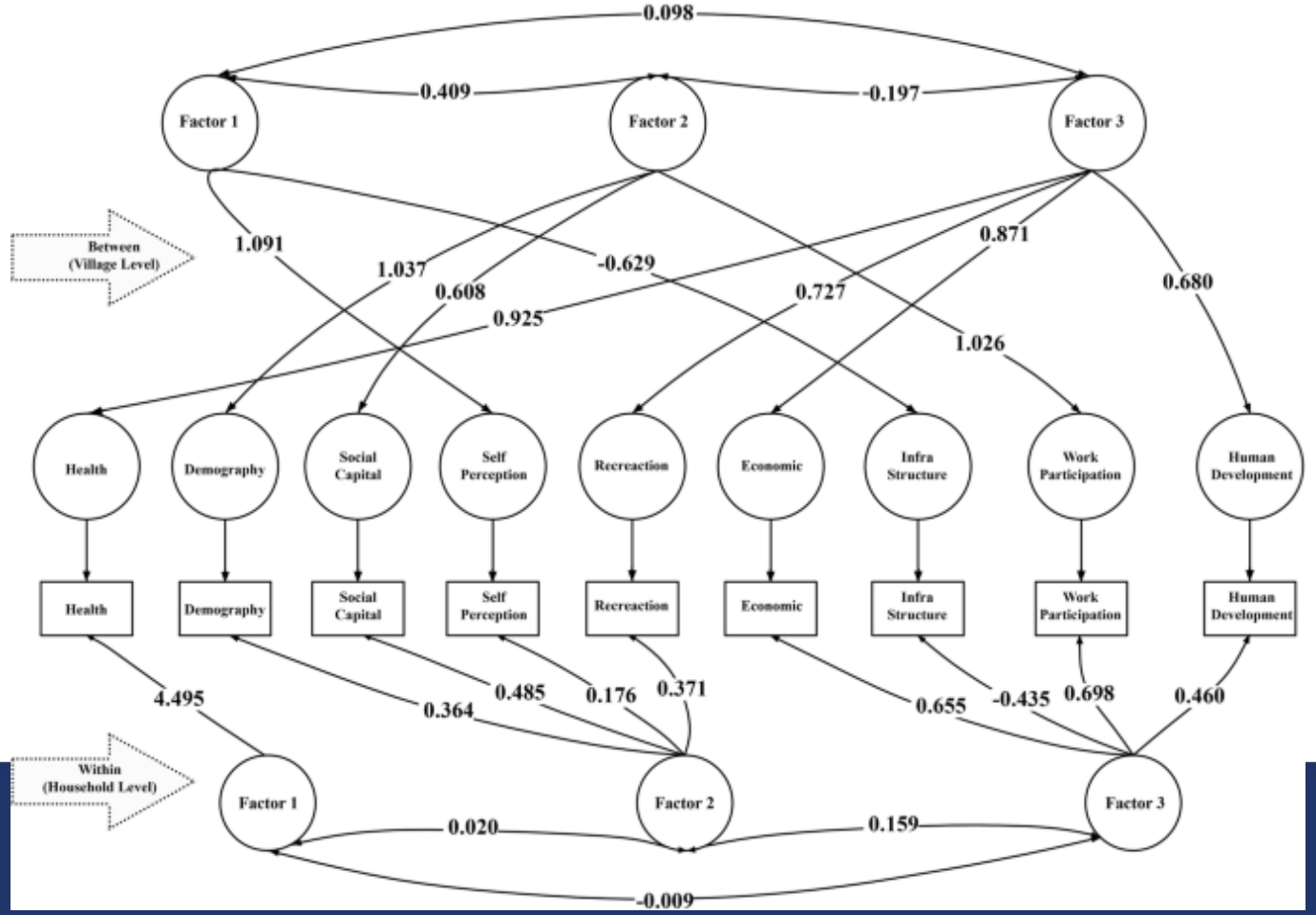
- The latent factor is **causing** some variation in ratings of items (measurements)

Latent Factor Analysis



Latent Factor Analysis

Quality of life of tribal communities in India



Latent Constructs

- Cognitive
  - General intelligence
  - Working memory
  - Processing speed
  - Executive function
- Personality
  - Openness
  - Neuroticism
- Self-efficacy
- Social Psych
  - Empathy
  - Right-wing Authoritarianism
- Clinical
  - Depression
  - Schizotypy
  - Impulsivity
- Developmental
  - Attachment style

What is latent factor analysis?

- **Latent variable** – unobserved construct that influences observed data
- **Factor analysis** – statistical method to identify underlying factors in a data set
- **Exploratory Factor Analysis (EFA)**
  - Finds latent structures without predefined hypotheses
- **Confirmatory Factor Analysis (CFA)**
  - Tests a specific hypotheses about factor structures

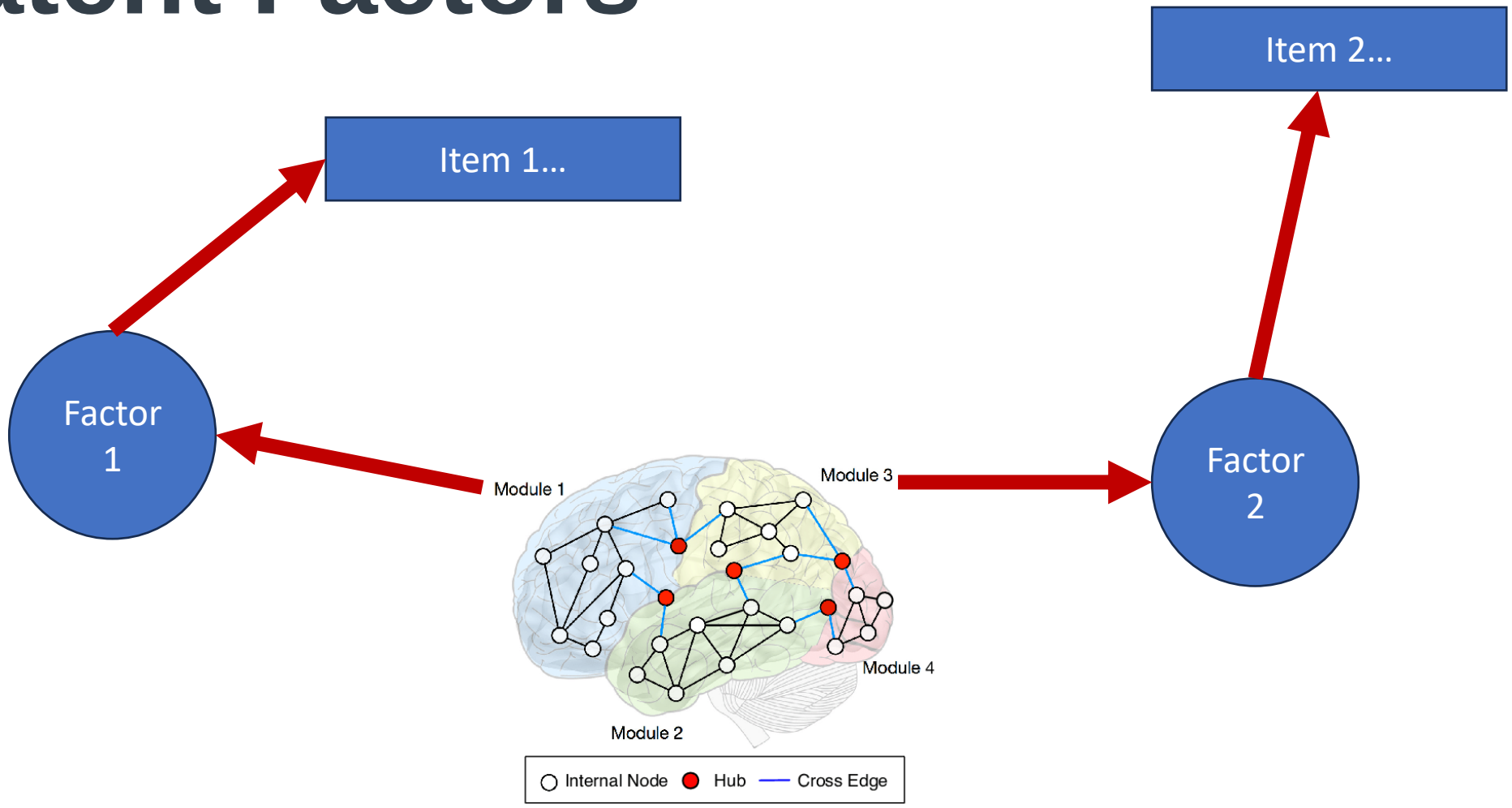
Statistical Foundations

- Observed variables have shared variance due to underlying latent factors
- Observed covariance matrix compared to model-implied covariance matrix for CFA
- For EFA, break into shared and unique variance for each variable
- Use eigenvalues to determine number of factors
- Get loadings and interpret

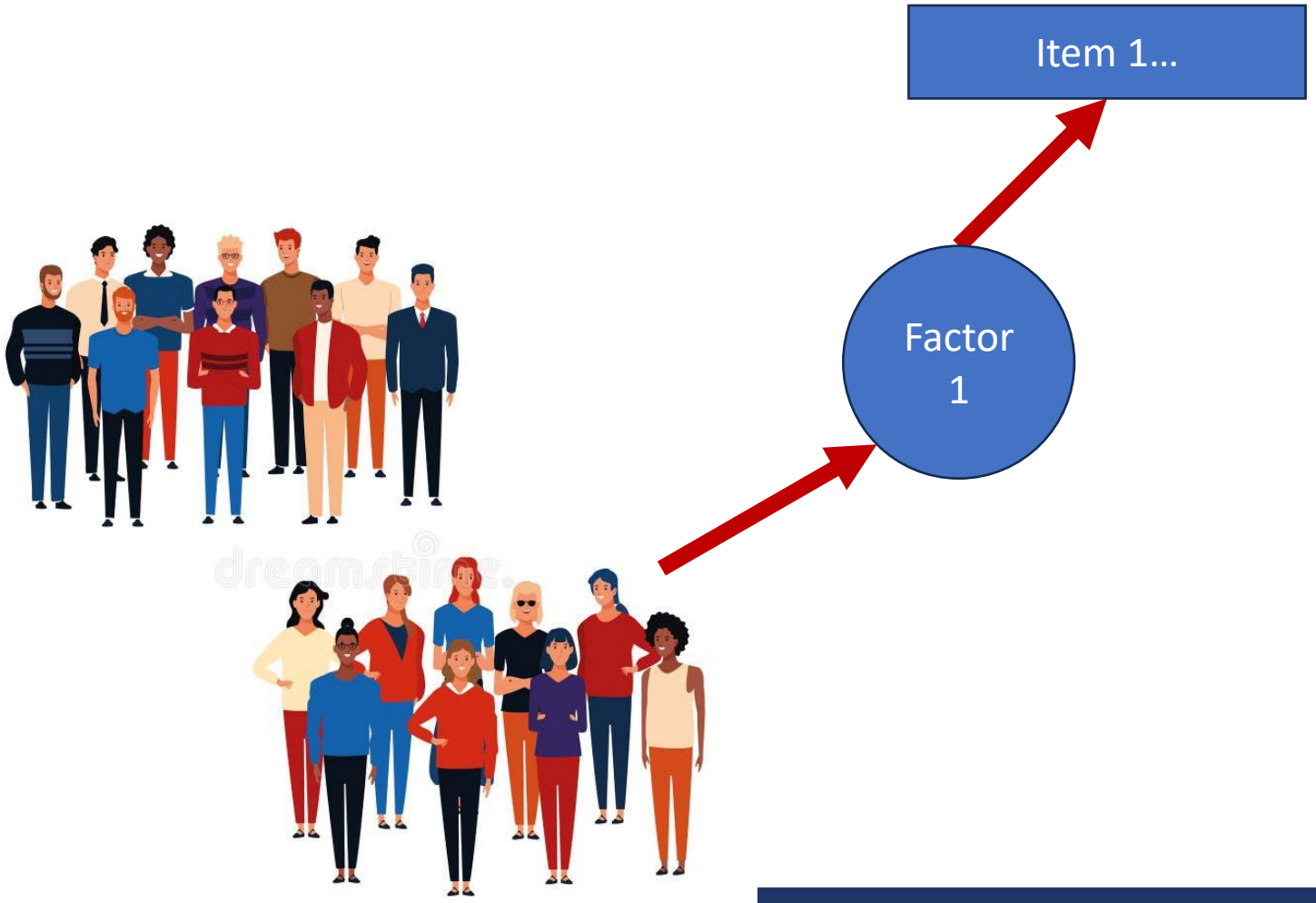
Model Fit

- Goodness-of-fit indices determine how well the model represents data:
  - Chi-square test: Compares observed vs. expected covariance matrices.
  - CFI (Comparative Fit Index): Should be ≥ 0.90 (good fit).
  - RMSEA (Root Mean Square Error of Approximation): Should be ≤ 0.08 (acceptable fit).
  - SRMR (Standardized Root Mean Square Residual): Should be ≤ 0.08.
- Factor loadings interpretation: Values ≥ 0.40 indicate a strong relationship between a variable and its factor.
- High loadings suggest good construct validity.

Latent Factors



Latent Factors



Moral Foundations Theory

Care

Justice

Loyalty

Authority

Sanctity

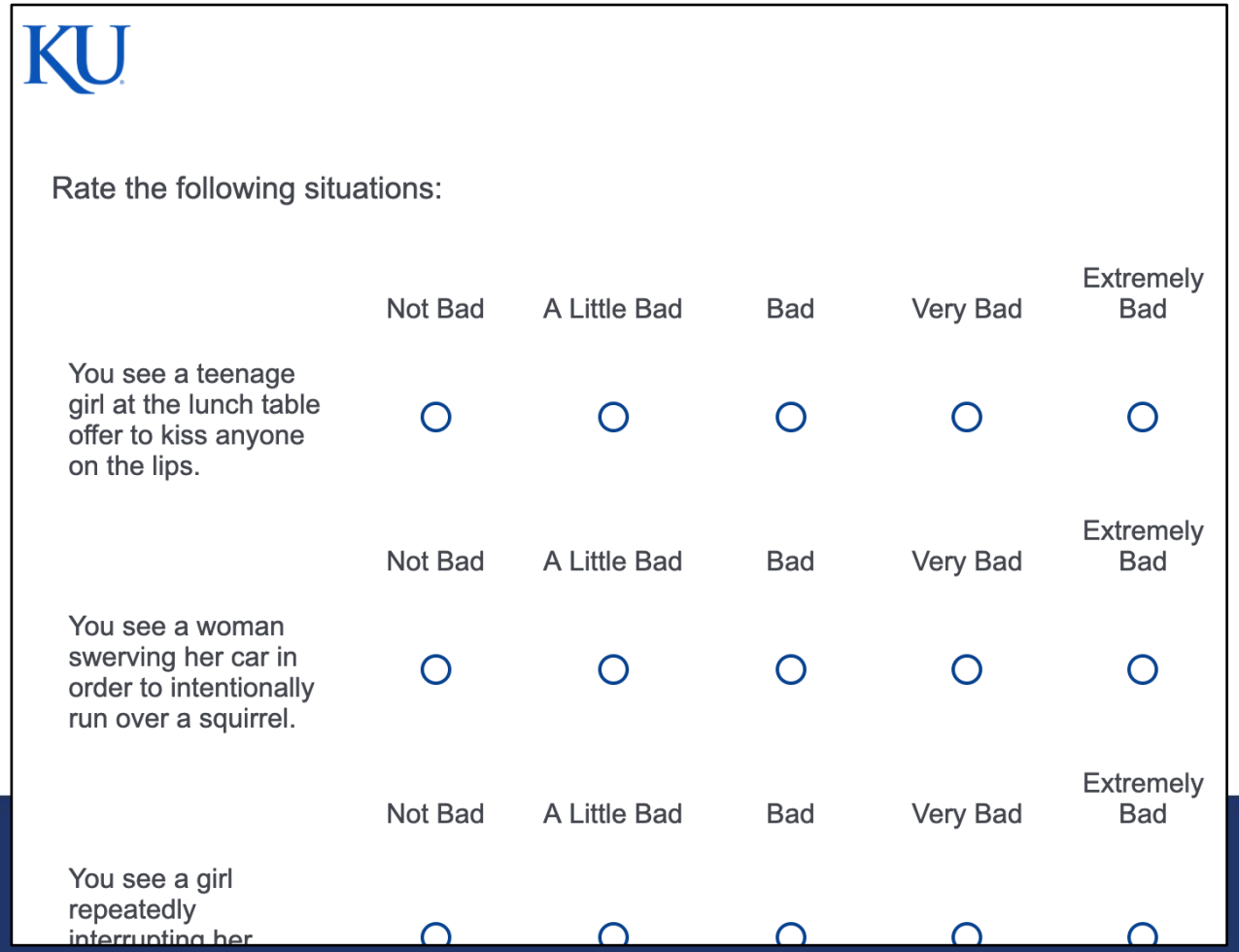
Autonomy

Moral Foundations Theory

- Previously validated
- Modified for use with children
- 48 items, 6 per factor

8 Factors

- Animal Physical
- Human Physical
- Human Emotional
- Justice
- Autonomy
- Authority
- Loyalty
- Sanctity



KU

Rate the following situations:

	Not Bad	A Little Bad	Bad	Very Bad	Extremely Bad
You see a teenage girl at the lunch table offer to kiss anyone on the lips.	○	○	○	○	○

	Not Bad	A Little Bad	Bad	Very Bad	Extremely Bad
You see a woman swerving her car in order to intentionally run over a squirrel.	○	○	○	○	○

	Not Bad	A Little Bad	Bad	Very Bad	Extremely Bad
You see a girl repeatedly interrupting her	○	○	○	○	○

Participants

- 3 suburban middle schools in Kansas
- n = 822
- M = 12.37 years old
- Predominantly Republican/Conservative counties

Demographics of Middle Schools Providing Participant Data

School	Demographics		
	Approx. Enrollment	Approx. Race	Subsidized Lunch
1	850	White 70% Black 5% Hispanic 20%	50%
2	750	White 55% Black 15% Hispanic 10%	60%
3	600	White 50% Black 15% Hispanic 20%	40%

Adult Factor Covariances, 48-item Subset, Clifford et al. (2015) Data

	Animal Physical	Human Physical	Human Emotional	Justice	Autonomy	Authority	Loyalty
Animal Physical	1						
Human Physical	0.696	1					
Human Emotional	0.623	0.782	1				
Justice	0.537	0.677	0.704	1			
Autonomy	0.594	0.663	0.691	0.547	1		
Authority	0.367	0.621	0.785	0.701	0.632	1	
Loyalty	0.365	0.497	0.712	0.627	0.567	**0.837**	1
Sanctity	0.495	0.639	0.762	0.718	0.547	**0.815**	0.766

All p-values < .001
*Covariances greater than **.8** in bold.*

n = 416
M = 34 years old

Latent Factor Covariance Table of 8-Factor Model, Adolescent Data

	Animal Physical	Human Physical	Human Emotional	Justice	Autonomy	Authority	Loyalty
Animal Physical	1						
Human Physical	0.758	1					
Human Emotional	0.717	**0.886**	1				
Justice	0.607	**0.858**	**0.903**	1			
Autonomy	0.708	0.695	**0.814**	0.729	1		
Authority	0.631	**0.835**	**0.906**	**0.973**	0.680	1	
Loyalty	0.439	0.560	0.597	0.654	0.626	0.706	1
Sanctity	0.489	0.732	**0.808**	**0.851**	0.572	**0.918**	0.640

All p-values < 0.001
*Covariances greater than **.8** in bold.*

Greater covariances between adolescent latent factors suggests a different factor structure than adults.

So an exploratory factor analysis was conducted…

n = 822
M = 12.37 years old

6-Factor EFA (Promax Rotation), Adolescent Data

Loading	ID	MFT Fact.	Item
			Factor 1
0.617	Q2_1	Hum. Emot.	...a girl laughing at another student forgetting her lines at a school play.
0.522	Q3_1	Hum. Emot.	...a woman commenting out loud about how bad another woman's hair looks.
0.802	Q1_3	Justice	...a student copying a classmate's answer sheet on a final exam.
0.470	Q2_3	Justice	...a runner taking a shortcut on the course during the marathon in order to win.
0.579	Q3_3	Justice	...someone cheating in a card game while playing with a group of strangers.
0.733	Q1_5	Authority	...a girl repeatedly interrupting her teacher as he explains instructions.
0.584	Q2_5	Authority	...a teenage girl coming home late and ignoring her parents' rules.
0.617	Q5_6	Authority	...a student say that her teacher is a fool during an afternoon class.
0.589	Q1_7	Sanctity	...a teenage girl at the lunch table offer to kiss anyone on the lips.
0.451	Q4_7	Sanctity	...a boy spit on the floor in the hallway.
			Factor 2
0.694	Q1_8	Anim. Phys.	...a man beating his pony with a whip for getting loose from its pen.
0.738	Q2_2	Anim. Phys.	...a woman throwing her cat across the room for scratching the furniture.
0.549	Q3_2	Anim. Phys.	...someone leaving his dog outside in the rain after it dug in the trash.
0.501	Q4_2	Anim. Phys.	...a boy throwing rocks at cows in a field.
0.565	Q5_2	Anim. Phys.	...a zoo trainer jabbing a dolphin to get it to entertain his customers.
0.433	Q3_8	Hum. Phys.	...a woman spanking her child with a spatula for getting bad grades in school.
			Factor 3
0.642	Q6_2	Hum. Phys.	...a boy placing a thumbtack sticking up on the chair of another student.
0.668	Q6_9	Hum. Phys.	...a girl whip a boy with a rope because she doesn't like him.
0.416	Q1_1	Hum. Emot.	...a teenage boy laughing at another boy with a disability.
0.448	Q4_3	Justice	...a referee intentionally making bad calls that help his favored team win.
0.683	Q6_3	Justice	...a teacher giving a bad grade to a student just because he dislikes him.
			Factor 4
0.490	Q2_4	Autonomy	...a mother telling her son that she is going to choose all of his friends.
0.497	Q3_4	Autonomy	...a man forbidding his wife to wear clothing that he has not first approved.
0.730	Q4_4	Autonomy	...a woman pressuring her daughter to become a famous evening news reporter.
0.666	Q6_4	Autonomy	...a mother forcing her daughter to enroll as a medical student in college.
			Factor 5
0.636	Q5_8	Sanctity	...a man blow his nose into his shirt.
0.634	Q6_8	Sanctity	...a woman not wash her hands after using a public toilet.
			Factor 6
0.443	Q1_6	Loyalty	...a coach celebrating with the other team's players who just won the game.
0.552	Q2_6	Loyalty	...a former US General saying publicly he would never buy any American product
0.500	Q3_6	Loyalty	...a mayor saying that the neighboring town is a much better town.
0.432	Q4_6	Loyalty	...a teacher publicly saying she hopes another school wins the math contest.

- Abstract, rule-based violations
- And/or unclear victim
- Or possibly culpable victim

Animal Physical

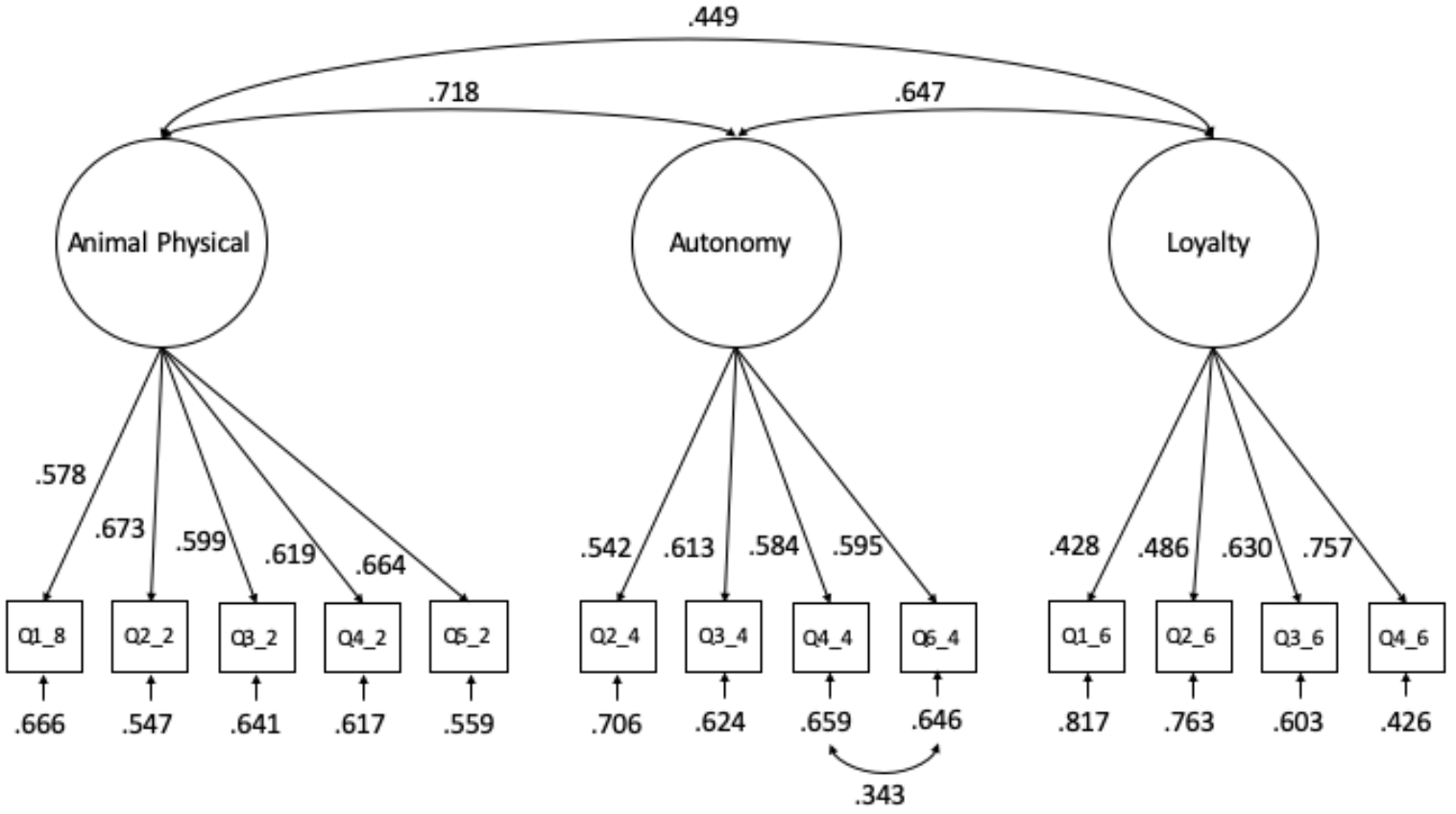
- Clearly innocent victim
- Obvious harm that is easy to empathize with

Autonomy

Sanctity

Loyalty

3-Factor Model



Measurement Invariance

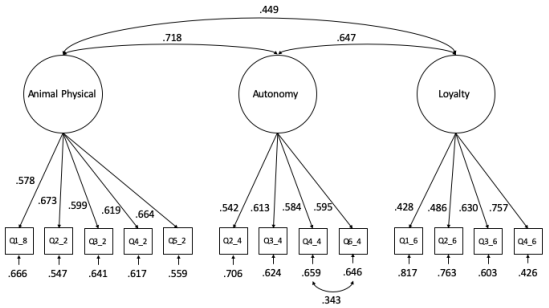
- Configural Invariance
  - Conceptual structure holds
- Metric Invariance
  - Ensure groups interpret factors similarly
- Scalar Invariance
  - Ensure latent mean comparisons are valid

RESEARCH ARTICLE

Age- and sex-based differences in the moral intuitions of American early adolescents

Brandon L. Bretl[1]* and Marlon Goering[2]

[1]University of Texas at Tyler, Tyler, Texas, USA and [2]University of Alabama at Birmingham, Birmingham, Alabama, USA
*Corresponding author. E-mail: BBRETL@uttyler.edu



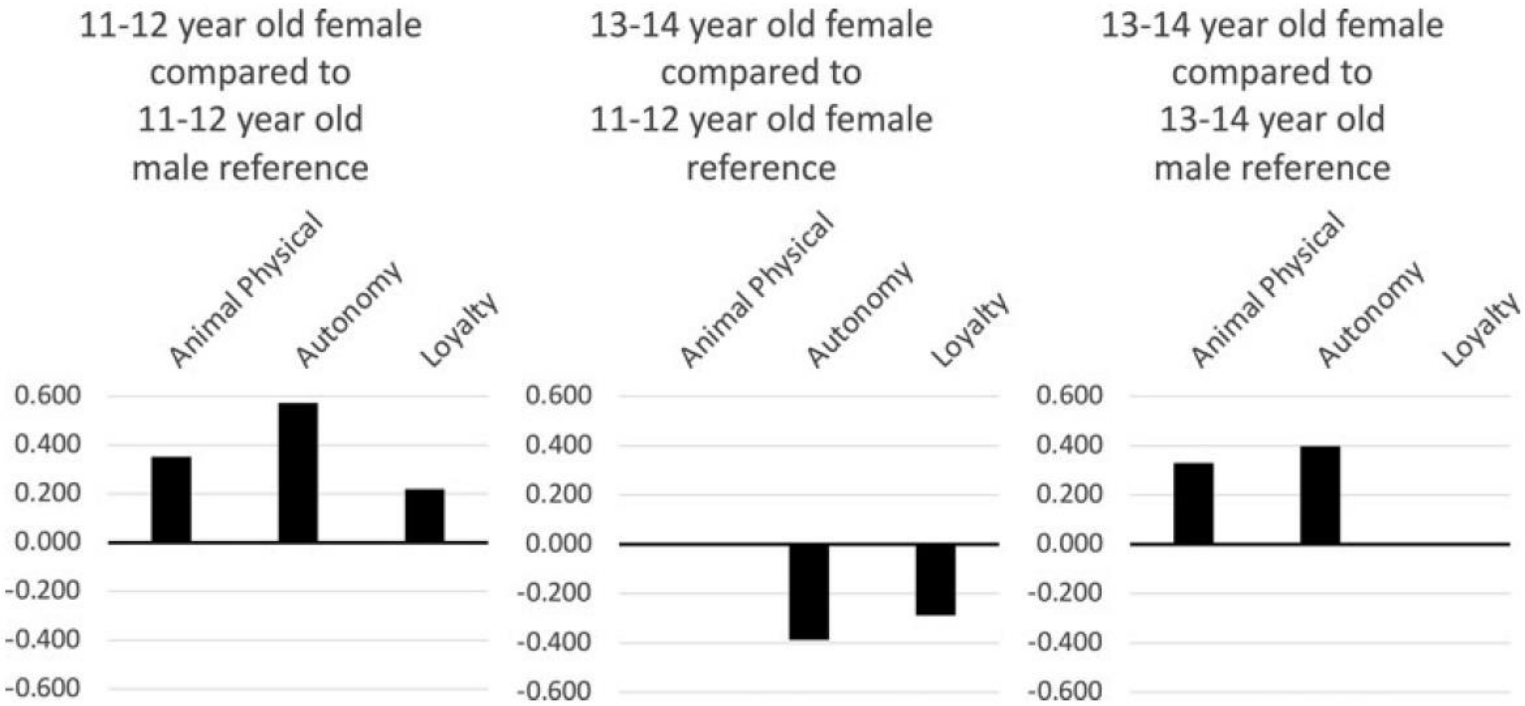


Figure 3. CFS-3 latent factor mean comparisons between age and sex groups. Reference group means constrained to zero. Graphs showing results at $p < 0.05$ significance level. No statistically significant results found between latent factor means of 11–12 year old males and 13–14 year old males.

Outside of social sciences…

Geology Example

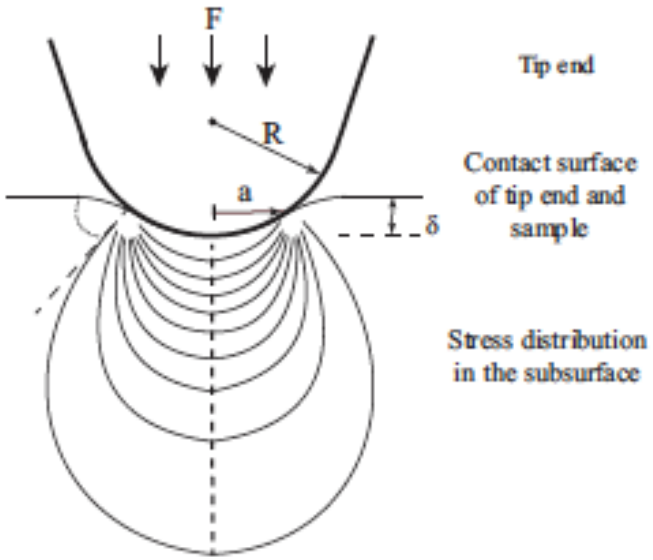


Figure (4) Stress distribution in the subsurface when force F is applied. The contact surface radius a is important for the stress distribution in the subsurface represented by the half circles.

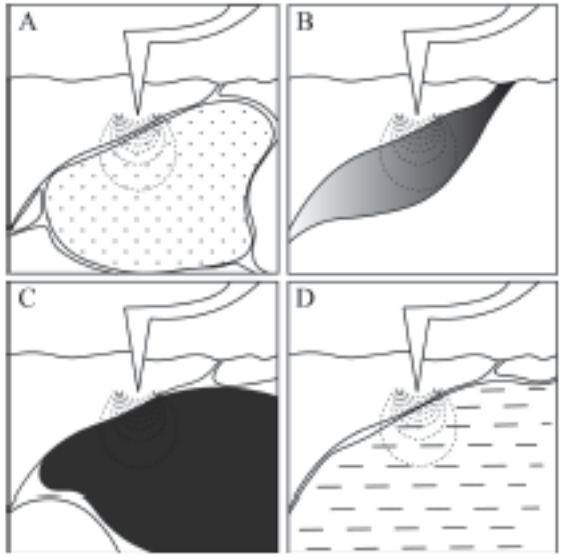


Figure (5) Influence of the subsurface showing a predicted varying result of Young's modulus for the same tested material on the surface. In the subsurface could be: a brittle quartz grain (A), pore space (B), organic matter (C) or a softer material such as clay (D).

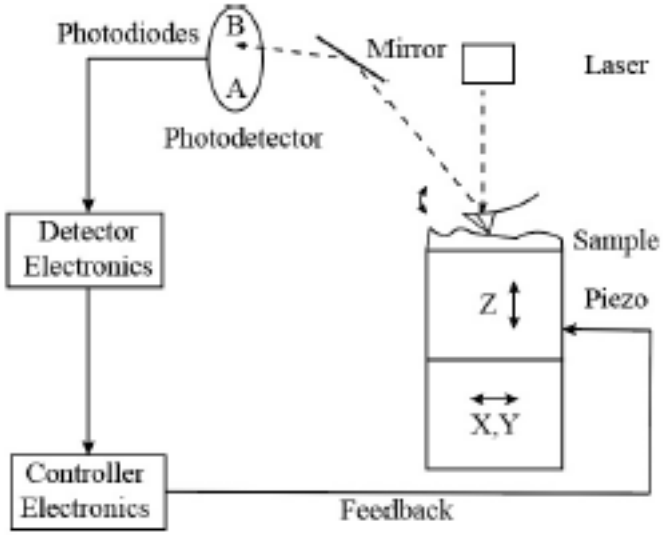


Figure (2) Schematics of AFM.

Geology EFA

4.1.3. Exploratory Factor Analysis

A preliminary parallel analysis suggested that a 6-factor model could parsimoniously estimate the variance/covariance matrices and explain 34% of the variance in the data set. This model was derived from the eigenvalues displayed in the scree plot (Figure 7).

Table (3) Factor loadings (regression slopes predicting indicators from latent variables). Higher loadings indicated a greater influence of that latent factor on the measurement outcome for that spot. Factor loadings smaller than 0.1 were excluded from the table.

Spot	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
1			0.956			
2						0.206
3	0.279				0.516	
4	0.277					0.265
5		1.015				
6	0.450			0.139	0.112	
7					0.141	0.516
8	0.460					
9				0.722		
10	0.100					
11	0.434			0.128		
12	0.534		0.060			0.103
13	0.275	0.121				
14	0.358				0.150	
15	0.330		0.127			0.109

Geology EFA

by a low strength material. The correlations between the factors increase with decreasing distance between the factors. Factor 4 and factor 6 show a higher correlation than factor 3 with factor 6. Factor 5 and factor 6 are expected to show a high correlation. However, these values show a weak correlation. Possible reasons for this poor agreement are the different origin for weaker strength, for example pore space or soft organic phase as both indicate low as values for Young's modulus.

The graphical interpretation of Young's moduli measurements reveal rock structures (Figure 8) and supports the factor model. Exploratory factor analysis has shown that it is possible to group the different tested spots. They are described by 6 factors where factor 1 is expected to be a brittle mineral phase with high strength whereas factor 5 and 6 revealed a softer rock component and pore space. The factors 2, 3 and 4 show a group of intermediate strength components. The literature has shown that a lower Young's modulus is indicates a soft organic phase (Young's modulus <25 GPa)[25], which is indicated by the factor 5 in this study. As the mean values of the spots from the factor 5 and 6 are higher than 25 GPa, it is expected that not only a soft organic phase was tested, but also the intergranular matrix. The intergranular matrix is described by values for Young's modulus of 25 GPa <E <50 GPa [60]. The factors 2, 3 and 4 describe the median intergranular matrix for the Eagle Ford shale. A stiff inorganic phase that contains isolated grains is described for a Young's modulus range of 50 GPa <E <100 GPa. Factor 1 represents this group of minerals. Overall eight spots out of 15 were represented by a stiff inorganic phase, which is also explained by the high weight percentages of calcite and quartz. Three spots were explained by an intergranular matrix and four spots by a soft organic phase and pore space. However, the correlations show that the groups are overlapping and mostly influenced by other factors as well. This study shows that Exploratory Factor Analysis is a powerful

Going even further…

Modification Indices

- Improving model fit by releasing constrained parameters so they can be estimated freely

Going even further…

You see…

…a girl repeatedly interrupting her teacher as he explains instructions.

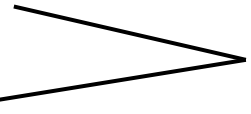
…a teenage girl coming home late and ignoring her parents' rules.

…a boy turning up the TV as his father talks about his military service.

…a teaching assistant talking back to the teacher in front of the classroom.

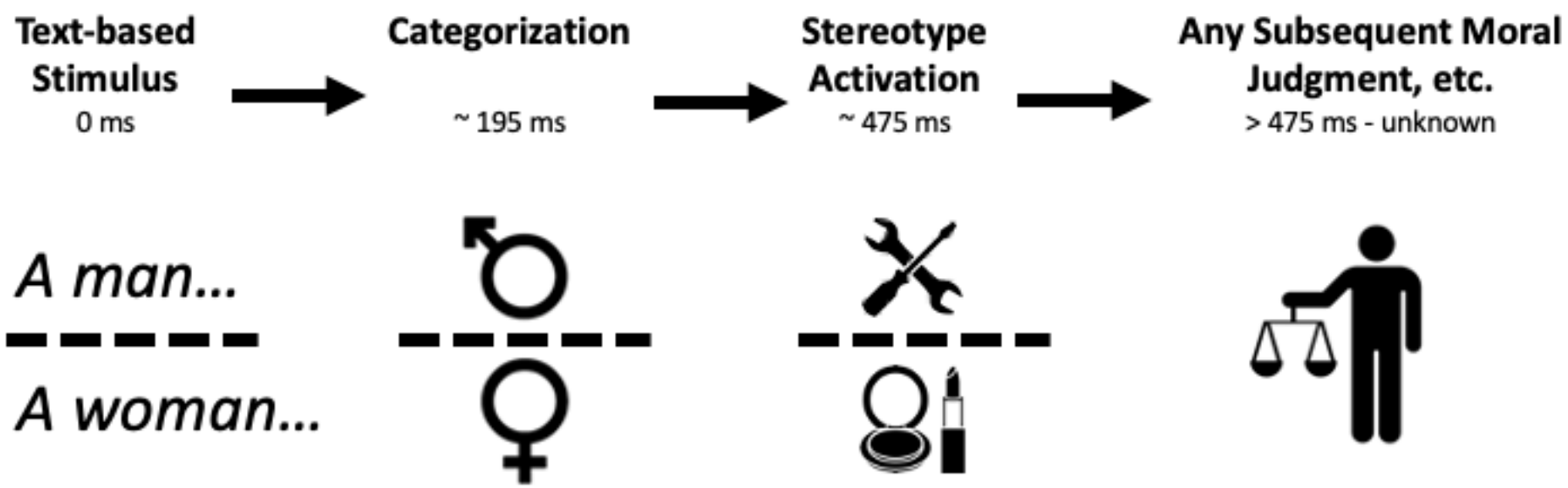
…a student say that her teacher is a fool during an afternoon class.

…a star player ignoring her coach's order to come to the bench during a game.



modification indices

Going even further…



The experiment…

Table 1. Four experimental conditions manipulating gender and first instance of gender information.

FN_1 Female noun *You see a girl . . . her . . .*	**FP_2** Female pronoun *You see a student . . . her . . .*
MN_3 Male noun *You see a boy . . . his . . .*	**MP_4** Male pronoun *You see a student . . . his . . .*

The results…

Table 2. Mean ratings for each condition by political party affiliation.

Violation Type	Conditions Compared	Mean Ratings	
		Republican	Democrat
Authority	FN_1, FP_2	3.15, 2.86 *	2.61, 2.71
	MN_3, MP_4	3.03, 3.07	2.78, 2.66
	FN_1, MN_3	3.15, 3.03	2.61, 2.78 *
Justice	FN_1, FP_2	3.61, 3.38	3.36, 3.31
	MN_3, MP_4	3.50, 3.58	3.47, 3.36
	FN_1, MN_3	3.61, 3.50	3.36, 3.47

* Significant difference at $p \leq .05$.

Some Practical Considerations

Some Practical Considerations

Type	Definition	Key Characteristics	Solution
Under-Identified	More free parameters than known values	- Model cannot be estimated (infinite solutions	- Add at least 3 items per factor - Fix one factor variance to 1 - Reduce unnecessary error correlations
Just-Identified	Equal number of known values & free parameters	- Model always fits perfectly ($\chi^2 = 0$, no df) - Fit indices cannot be tested	- Add more indicators per factor - Introduce higher-order structures if needed
Over-Identified	More known values than free parameters	- Allows statistical testing of model fit (χ^2, RMSEA, CFI, etc.) - Required for robust CFA models	- Ensure **≥4 indicators per factor** - Constrain parameters where necessary - Avoid excessive error covariances

Some Practical Considerations

- Need large sample sizes
- 4 items per factor
- For scale development, create 6-12 items per factor, keep the 4 best.

Model Type	Recommend N
1-2 factors, 3-4 indicators each	200-300
3-4 factors, moderate loadings	300-500
Complex models (e.g., hierarchical CFA, cross-loadings)	500+
Very large, multi-group CFA (e.g., testing invariance across groups)	600-1000+

Factor Loadings (Standardized)	Recommended N
≥ 0.80 (strong)	100-200
~0.60 - 0.80 (moderate)	300-400
≤ 0.40 (weak)	500+ (or reconsider items)

Tools

- R and R studio
  - Psych, lavaan, and sem packages
- Confirmatory Factor Analysis for Applied Research
  - Timothy A. Brown (2015)

Thank You

BBRETL@uttyler.edu