# MATH 5351 Probability

**Meeting Times:** 10:10-11:05 pm MWF in RBN 3040.

**Last day to withdraw:** Monday, November 4, 2024

**Instructor:** Nathan Smith

**Office:** RBN 4007

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**Office Hours:** Tentatively 1:30 - 2:30 MW, 9:00-10:00 F with other times available by appointment.

**Text:** We will be using free online materials rather than a purchased text. We'll have two main sources, the book *Basic Probability Theory* by Robert Ash (henceforth BPT) and the book *A Modern Introduction to Probability and Statistics (Understanding How and Why)* by Deeking, Kraaikamp, Lopuhaä, and Meester (henceforth MIPS). There are pdf versions of each book uploaded to Canvas. Each and every choice of a textbook would come with its own flaws, but the price is unbeatable here, so we're going to accept the flaws of these texts and make the best of things.

**Course Topics:** Set theoretic probability, discrete and continuous random variables, expectation and varaince, the central limit theorem, a little statistical inference.

**Student Learning Outcomes:** By the end of the course students should be able to:

- Define and use the basic concepts from probability theory, including random variables, distribution and density functions, conditional probability, moments, expectations, generating functions, and the central limit theorem.
- Identify, give density functions for, and apply the standard discrete and continuous random variables and random variables derived from these.
- Apply the above to basic statistical inference.

**Grading:** There will be two tests as well as a final exam. Each will count one fifth of your overall course grade. You will each complete a project (we'll begin about half-way through the course and it will be due toward the end of the course) which will count for one fifth of your overall course grade. The remaining fifth of your course grade will be based on homework and classwork.

**Project:** You will choose/be assigned a topic that is related to probability, but is outside the scope of the course proper. I will distribute a list of possible topics when the project is assigned. The "project" is to write a paper about your topic. I know that the first question is "how many pages does it have to be" and that is difficult to answer with an exact number. It seems to me that it would be difficult to do any of the topics justice in under 10-15 pages or

so, but maybe you can write in a sufficiently terse Lemma/Proof/Theorem/Proof style and make it work. While there is no need for grandiose verbosity, thoroughness is the important thing here, and it will be necessary to include sufficient examples to illustrate your topic. You should write your paper with your classmates as the intended audience. In fact two members of that very audience will read and evaluate your paper, and you will, in turn, read and evaluate two of their papers. I will, of course, read and evaluate everyone's paper myself as well.

**Student Academic Conduct:** It is your responsibility to learn the material in this course for your own benefit. You should not let this discourage you from working together on your homework but in the end what you turn in should reflect your understanding, not just be copied from someone else. *During the tests and the final exam a code of honor will apply under which students are to work alone and neither give help to others nor receive help from any sources.* Students are also expected to help enforce this code. Students are encouraged to obtain a copy of *A Student Guide to Conduct and Discipline at UT Tyler*, available in the Office of Student Affairs.

**Artificial Intelligence:** UT Tyler is committed to exploring and using artificial intelligence (AI) tools as appropriate for the discipline and task undertaken. We encourage discussing AI tools' ethical, societal, philosophical, and disciplinary implications. All uses of AI should be acknowledged as this aligns with our commitment to honor and integrity, as noted in UT Tyler's Honor Code. Faculty and students must not use protected information, data, or copyrighted materials when using any AI tool. Additionally, users should be aware that AI tools rely on predictive models to generate content that may appear correct but is sometimes shown to be incomplete, inaccurate, taken without attribution from other sources, and/or biased. Consequently, an AI tool should not be considered a substitute for traditional approaches to research. You are ultimately responsible for the quality and content of the information you submit. Misusing AI tools that violate the guidelines specified for this course (see below) is considered a breach of academic integrity. The student will be subject to disciplinary actions as outlined in UT Tyler's Academic Integrity Policy.

For this course, you may not use AI tools to produce anything turned in for a grade.

**University Policies:** For University policies concerning Students' Rights and Responsibilities, Grade Replacement/Forgiveness, State-Mandated Corse Drop Policy, Disability Services, Student Absence due to Religious Observance, Student Absence for University-Sponsored Events and Activities, and the Social Security and FERPA Statement please see: *http://www.uttyler.edu/academicaffairs/files/syllabuspolicy.pdf.*

# Course Outline

We'll have two main sources, the book *Basic Probability Theory* by Robert Ash (henceforth BPT) and the book *A Modern Introduction to Probability and Statistics (Understanding How and Why)* by Deeking, Kraaikamp, Lopuhaä, and Meester (henceforth MIPS). There are pdf versions of each book uploaded to Canvas. Each and every choice of a textbook would come with its own flaws, but the price is unbeatable here, so we're going to accept the flaws of these texts and make the best of things.

A constant source of friction with pdf books, though not enough to offset the low, low price, is page numbering. In an effort to be consistent I will always refer to page numbers as they are printed in the top left/right corners of the pages in the pdf book, i.e. the original book pages. Unfortunately these are not the page numbers that are likely to be listed as page numbers of the pdf file in your viewer. For the BPT book, the pdf page numbers are off by 10, so if you want to look at book page 181, for example, which I will always refer to as "page 181," you'll need to go to pdf page 191. For the MIPS book things appear to be more complicated, as some (likely originally blank) pages in the book, such as those between chapters, seem to have been omitted, so that the distance between book page number and pdf page number drifts, starting off such that book page 1 is pdf page 14, and ending up with book page 473 being pdf page 472. If you are enrolled in a graduate course in probability you will be able to figure this out and make things work.

1. Sigma algebras. BPT 1.2

    - Review of set nonsense, unions, intersections, complements, subtraction, cartesian products, disjointness (mutually exclusive)
    - distributive properties, DeMorgan's laws finite and infinite,
    - sigma field $\mathcal{B}$ on $\Omega$ has $\Omega$ as a member and is closed under complements and countable unions.

2. Probability spaces. BPT 1.2 and MIPS 2.3, 2.4

    - Probability function $\mathcal{B} \to [0, 1]$.
    - Outcomes, events, etc.
    - Examples of sample spaces finite, infinite, and product spaces

3. Combinatorics stuff BPT 1.4

    - Factorials, permutations, combinations
    - order matters vs. repeats allowed square formulas

4. Conditional probabability and independence, Bayes BPT 1.5, 1.6, MIPS ch 3

- conditional probability
- law of total probability
- Bayes' rule
- independence

5. Random variables BPT ch 2, MIPS ch 4 and 5 and part of 8

   - Definitions and $F$
   - Discrete random variables and what $f$ means here. Binomial, geometric, poisson
   - Continuous random variables and what $f$ means here. Uniform, exponential, pareto, normal, gamma
   - Functions of r.v., change of variables
   - random vectors and joint densities
   - multivariable change of variables

6. Simulations, MIPS ch 6

7. Expectation BPT ch 3, MIPS ch 7 and 10

   - Definition and linearity
   - Variance and Standard Deviation
   - Covariance and Correlation
   - Markov and Chebychev inequalities, WLLN
   - Moment generating functions

8. Conditional densities and conditional expectation

9. sequences of random variables BPT ch 6

   - Convergence in distribution
   - Convergence in probability
   - Central Limit Theorem