Syllabus for the M.S. Comprehensive Examination in Real Analysis

The University of Texas at Tyler
Department of Mathematics

For each topic listed below, it is expected that the student will know the pertinent definitions, propositions, theorems, corollaries, etc. It is also expected that the student be able to apply these definitions, propositions, and theorems to solve related problems. Part I below gives the topics corresponding to the Low Pass section of the exam and Part II corresponds to the High Pass section of the exam.

I. Basic Analysis (Undergraduate Prerequisites)
- a. Convergence of Sequences of Real Numbers
    - i. Prove, from definition, convergence for a specific sequence of real numbers.
    - ii. Prove, from definition, basic limit laws concerning sequences of real numbers.
    - iii. Divergence (to infinity or otherwise)
    - iv. Subsequences
    - v. Cauchy sequences of real numbers
    - vi. Convergence criteria (e.g. Monotone Convergence Theorem, The Squeeze Law, Bolzano-Weierstrass Theorem, Cauchy Convergence Criterion)
    - vii. The limit superior and limit inferior and its relation to convergence of sequences of real numbers
- b. Sequences of functions
    - i. Pointwise convergence
    - ii. Uniform convergence
- c. Limits of Functions
    - i. Prove, from definition, convergence for a specific function
    - ii. Prove, from definition, basic limit laws concerning limits of functions
    - iii. The connection of limits of functions to limits of sequences
    - iv. Convergence criteria (how might these be inherited from sequences?)
    - v. One-sided limits
    - vi. Divergence (to infinity or otherwise)
    - vii. Limits at infinity
- d. Continuity
    - i. Prove, from definition, continuity of a function at a point
    - ii. The connection of continuity of a function at a point to limits of sequences
    - iii. Continuity of combinations of functions
    - iv. Maximum and Minimum Value Theorem (of a continuous function on a closed interval)
    - v. The Intermediate Value Theorem
- e. Derivatives
    - i. Calculate a derivative from the definition
    - ii. Increasing/decreasing functions
    - iii. Critical points and local extrema
    - iv. Differentiability implies continuity
    - v. The Mean Value Theorem and its important corollaries (e.g. two functions whose derivatives are equal differ by at most a constant)
    - vi. The Cauchy Mean Value Theorem
    - vii. L'Hospital's Rule
- f. The Riemann Integral
    - i. The definition in terms of upper and lower integrals
    - ii. Calculation of integrals from uniform partitions (low-order polynomials in particular)
    - iii. Integrability criteria (monotone functions, continuous functions)

iv. Mean-Value Theorem for Integrals
v. The Fundamental Theorem of Calculus

Measure Theory (First semester graduate analysis)

g. Borel Sets
h. Lebesgue Measure
   i. Outer measure
   ii. Measurable sets and Lebesgue measure
   iii. Nonmeasurable sets
   iv. Measurable functions
   v. Littlewood's three principles
i. The Lebesgue Integral
   i. The Lebesgue Integral of a Bounded Function over a set of Finite Measure
      1. Riemann integrability implies Lebesgue integrability
      2. The Bounded Convergence Theorem
   ii. The Lebesgue integral of a nonnegative function
      1. Monotone Convergence Theorem
   iii. The general Lebesgue integral
      1. Lebesgue Convergence Theorem
j. Differentiation and Integration
   i. Vitali Covering Theorem
   ii. Derivatives and differentiability
   iii. Measurability of the derivative
   iv. Functions of bounded variation
   v. Differentiability of the Integral (i.e. the Lebesgue version of the Fundamental Theorem of Calculus)
   vi. Absolute continuity

II. Advanced Topics

a. Convex functions
b. L^p and l^p spaces
c. Minkowski and Holder Inequalities
d. Convergence and completeness in L^p (and l^p)
   i. Banach space
   ii. Absolutely summable series
   iii. Riesz-Fischer Theorem (for L^p and l^p)
e. Approximation in L^p.
f. Bounded linear functionals on L^p.
   i. Riesz Represenation Theorem
   ii. This topic can be adapted to l^p.
g. Further topics may be taken from the following categories, depending on the discretion of the examiners)
   i. Metric spaces
   ii. General topology
   iii. General Banach spaces
   iv. General measure and integration theory
   v. Measure and outer measure