Algebra M.S. Exam Topics

Below is an listing of "standard topics," and any exam is likely to contain many questions on topics drawn from this list. The student should be aware that exam topics will ultimately be determined by the instructor for the course and the examination committee.

1. Groups: Topics to include subgroups, homomorphisms, normal subgroups, isomorphisms, automorphisms, and quotients. Lagrange's theorem and cosets. The isomorphism theorems. Group actions, centralizers, normalizers, the class equation. Permutation groups and permutation representations. The sylow theorems. Direct and semidirect products, finitely generated abelian groups. Direct and semidirect products, finitely generated abelian groups. Generators and relations, free groups, nilpotent and solvable groups.

2. Rings: Topics to include ideals, prime ideals, maximal ideals, homomorphisms, quotient rings, the isomorphism theorems for rings. Euclidean domains, principal ideal domains, unique factorization domains, polynomial rings, local rings, rings of quotients.

3. Modules: Topics to include submodules and homomorphisms, annihilators, quotients, direct sums, direct products, tensor products. Modules over a principal ideal domain, rational canonical form. Exact sequences, free modules, projective and injective modules, diagram chasing, the five lemma.

4. Fields: Topics to include field extensions, finite fields, algebraic extensions, algebraic closures, separable and normal extensions, splitting fields, and Galois theory. Vector spaces, bases, subspaces, dimension, matrices and linear maps, characteristic polynomials and the Cayley-Hamilton theorem, eigenvalues, diagonalization.