MATH 5342 – Real Analysis II

Spring 2023

Instructor: Dr. Scott M. LaLonde
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Office hours: Tuesdays and Thursdays 1:00–2:30 P.M, or by appointment.

Scheduled lectures:
Section 001
MWF, 9:05 – 10:00 A.M.
Location: RBN 4039

Course Webpage: All course information and documents will be available on Canvas.

Textbooks: In addition to my own notes (which will be available on Canvas), the official textbook for the course is

Measure, Integration, and Real Analysis by Sheldon Axler (ISBN: 978-3-030-33142-9).

This book is freely available on the author's website (`https://measure.axler.net`). It is also published by Springer, so it is possible to purchase a physical copy if you so desire.

Prerequisites: A grade of B or better in MATH 5341 (Real Analysis I) or its equivalent.

Course Description

Topics may include general topology, Banach spaces, generalized measure and integration, or related topics. Course may be repeated when content changes.

Student Learning Outcomes

Upon completion of this course, students should be able to do the following:

- Formulate correct, rigorous mathematical proofs of results in the realm of real analysis.
- Articulate relevant mathematical ideas correctly and precisely, both verbally and in writing.
- Outline the construction of Lebesgue measure and the Lebesgue integral on $\mathbf{R}$.
- Summarize the construction of the integral associated to an abstract measure space.
- Understand and apply the Monotone Convergence Theorem, Dominated Convergence Theorem, and Fatou's Lemma for the Lebesgue integral, and construct examples to illustrate the importance of each theorem.
- Understand and apply basic properties of Banach and Hilbert spaces, and be able to work with basic examples of each (particularly the L^p spaces).

- Understand and apply some classical results from functional analysis, including the Hahn-Banach Theorem, the Open Mapping Theorem, the Closed Graph Theorem, and the Principle of Uniform Boundedness.
- Work with bounded linear functionals and bounded operators on Banach and Hilbert spaces.

Assignments and Grading

Homework

Homework will be assigned more or less weekly. Abstract mathematics is best learned through practice, so it is imperative that you make an honest effort to complete each assigned problem. Homework assignments need to be written legibly or typed, and **all proofs must be written in complete sentences.** I reserve the right to reject any papers that are illegible. Homework will generally be due on Fridays by 5 P.M., and you will submit your work via Canvas.

Each assignment will consist largely of proofs, though there will also be computational problems where you will work with specific examples. Some problems will come from the textbook, but I will also take many of the problems from other sources. I will divide each assignment into three sections according to the difficulty of the problems:

- **Easy:** These are basic problems designed to check your understanding of the key concepts.
- **Medium:** These problems will require you to apply the concepts you have learned in class, usually to prove new facts. The majority of the homework problems will fall in this category.
- **Hard:** These problems are fairly difficult. They likely require some creativity, or you may need to explore concepts beyond what we have done in class.

You have hopefully come to learn that an attempt at writing a mathematical proof either ends with a correct proof, or it does not. However, I will grade each problem on a scale of 0 to 4 to allow for partial credit. The score will depend on both content and presentation as follows:

- **4:** You have constructed a correct proof of the given statement, and it is written clearly, coherently, and in complete sentences.
- **3:** Your solution is mostly correct, but there are some small defects that keep your argument from being completely airtight. It is also written clearly and in complete sentences.
- **2:** You are headed in the right direction, but there are fundamental flaws in the argument or exposition. Your proof is partially correct, or you've cut corners in the written presentation.
- **1:** Your proof is fundamentally flawed and/or poorly written.
- **0:** Your proof is completely incorrect or incoherent, or you did not make a reasonable attempt at solving the problem.

Exams

There will be two exams during the semester, as well as a comprehensive final exam. The exams will be given in a sit-down, timed format, but I would like to hold the exams outside of class in order to allow you more time to work on them. We will discuss this early in the semester, and I will announce the exam schedule afterward. Tentatively, the dates I have in mind are as follows:

- Exam 1: Week of February 20
- Exam 2: Week of April 3
- Final Exam: April 24, 8:00–10:00 A.M.

Each exam will consist of a combination of conceptual questions, computational questions, and proofs. We will discuss the content in more detail prior to each exam. All exams are closed book, with **no books, notes, or calculators allowed**. No help will be given or received.

Grading

Your grade will be computed as follows. The rubric for assigning final letter grades will be *no more harsh* than the given scale.

Assignment	Total %
Homework	25
Exam 1	25
Exam 2	25
Final exam	25
Total	100

Numerical	Letter
85 – 100	A
70 – 84	B
55 – 69	C
Below 55	F

Course Policies

Canvas

You must activate your Canvas account and check it regularly. You can activate your account and log in at `https://www.uttyler.edu/canvas`. If you are registered for the course, then you should already have access to the Canvas page. All homework assignments, announcements, and important documents will be posted there.

Collaboration, Plagiarism, and Academic Dishonesty

I encourage you to talk to your classmates when studying and working on homework assignments. When learning abstract mathematics, it is extremely helpful to discuss ideas with others, and it can be easier to discern what one does and does not understand when trying to explain things to others. Therefore, collaboration is an indispensable learning tool. However, any work you submit must represent your own effort. Keep the following guidelines in mind when working on homework:

- The solutions that you turn in to me should be written up by you in your own words. It is fine (and encouraged) to discuss ideas with others, but I want each person to think individually about how to put those ideas down on paper.

- If you have worked with others on a particular problem, say so when you write up your solution. If you got a particular idea from someone else, give them the appropriate credit.

In summary, I encourage you to work on homework together, but I do not want you to write up complete proofs as a group—this should be done individually.

Finally, do not present solutions that you have found in other textbooks or on the internet as your own. Aside from committing plagiarism, doing so defeats the purpose of the homework (which is to learn the material through practice). If I determine that you have submitted work that is not your own, I will prosecute plagiarism and academic dishonesty to the fullest possible extent.

Office Hours

I have regularly scheduled office hours, which are set aside as time for you to come talk to me about the course. Don't be afraid to come ask me questions when you are studying or working on homework. This is a difficult course, and it helps greatly to talk to someone about the material. If you are unable to attend my usual office hours, you can always set up an appointment via email.

Attendance

I expect you to attend class every day. Attendance is not officially part of your grade, but it will be very hard for you to succeed in this course if you are repeatedly absent from class. In particular, I will not always cover topics from the same perspective as the textbook, and we may even cover some things in class that are not mentioned in the book. Additionally, this course is intended to build mathematical maturity through reading and writing proofs, and lectures and in-class discussions will help greatly with this.

If you do miss a class, you are responsible for the material that was covered that day. You are also responsible for any announcements made in class.

Make-up Policy

Make-ups will be granted for for cases of severe illness, excused absences that are required as part of a UT Tyler obligation, or religious observances. You must notify me ahead of time and provide appropriate documentation. Other makeups will be granted at the discretion of the instructor. Makeups will not be granted after the fact under any circumstances.

Cell Phones, Calculators, and Electronic Devices

When class is about to begin, place any electronic devices (e.g. cell phones) in silent mode and put them out of sight. You may use a laptop or tablet to take notes if you wish. If you are using such devices for other purposes, I will ask you to put them away.

COVID-19

Please continue to follow UT Tyler and CDC guidelines regarding COVID-19. You can find the university's current guidance at the following link:

`https://www.uttyler.edu/coronavirus/`

You should continue to take appropriate precautions to avoid exposing yourself and others to COVID-19, influenza, or other communicable diseases. In particular, please stay home if you are not feeling well or you are exhibiting symptoms consistent with COVID-19 or the flu.

Changes to Syllabus

I reserve the right to make changes to the syllabus during the semester. I will announce any changes to course policies in class and post an updated version of the syllabus to Canvas.

Important Dates

January 9: Classes begin.
January 16: Martin Luther King, Jr. holiday. No class.
January 23: Census date. Last day to change schedule or file for grade replacement.
March 13–18: Spring break. No classes.
March 23: Last day to withdraw.
April 24: Final exam.

University Policies

Information on University policies concerning the following topics:

- UT Tyler Honor Code
- Students Rights and Responsibilities
- Campus Carry
- UT Tyler Tobacco-Free Policy
- Grade Replacement/Forgiveness and Census Date
- State-Mandated Course Drop Policy
- Student Accessibility and Resources
- Student Absence due to Religious Observance
- Student Absence for University-Sponsored Events and Activities
- Social Security and FERPA Statement
- Emergency Exits and Evacuation
- Student Standards of Academic Conduct
- UT Tyler Resources for Students

can be found at

`https://www.uttyler.edu/academic-affairs/files/syllabuspolicy.pdf`

Tentative Weekly Schedule

This schedule is very likely to change as we move through the semester. Topics marked with an asterisk (*) are optional and will be covered as time permits.

Week	Dates	Topics covered
1	1/9–1/13	Lebesgue measure on the real line.
2	1/16–1/20	Properties of Lebesgue measure; Non-measurable sets.
3	1/23–1/27	Measurable functions.
4	1/30–2/3	The Lebesgue integral on $\mathbf{R}$.
5	2/6–2/10	Convergence theorems for the Lebesgue integral.
6	2/13–2/17	The theorems of Egorov and Lusin. Littlewood's three principles of real analysis.
7	2/20–2/24	Abstract measure spaces and integration.
8	2/27–3/3	The convergence theorems for general measures. Modes of convergence.
9	3/6–3/10	Product measures and the theorems of Fubini and Tonelli.
10	3/13–3/17	Spring break – no classes.
11	3/20–3/24	Signed and complex measures, differentiation, and the Radon-Nikodym theorem.
12	3/27–3/31	Basic properties of L^p spaces. The Riesz-Fischer theorem.
13	4/3–4/7	Introduction to Banach and Hilbert spaces. Bounded operators.
14	4/10–4/14	Linear functionals, dual spaces, and the Riesz representation theorem.
15	4/17–4/21	The Hahn-Banach theorem and its applications. The Open Mapping Theorem, Closed Graph Theorem, and the Principle of Uniform Boundedness.