UTTyler Center for Health Informatics & Analytics

MACHINE LEARNING: REGRESSION

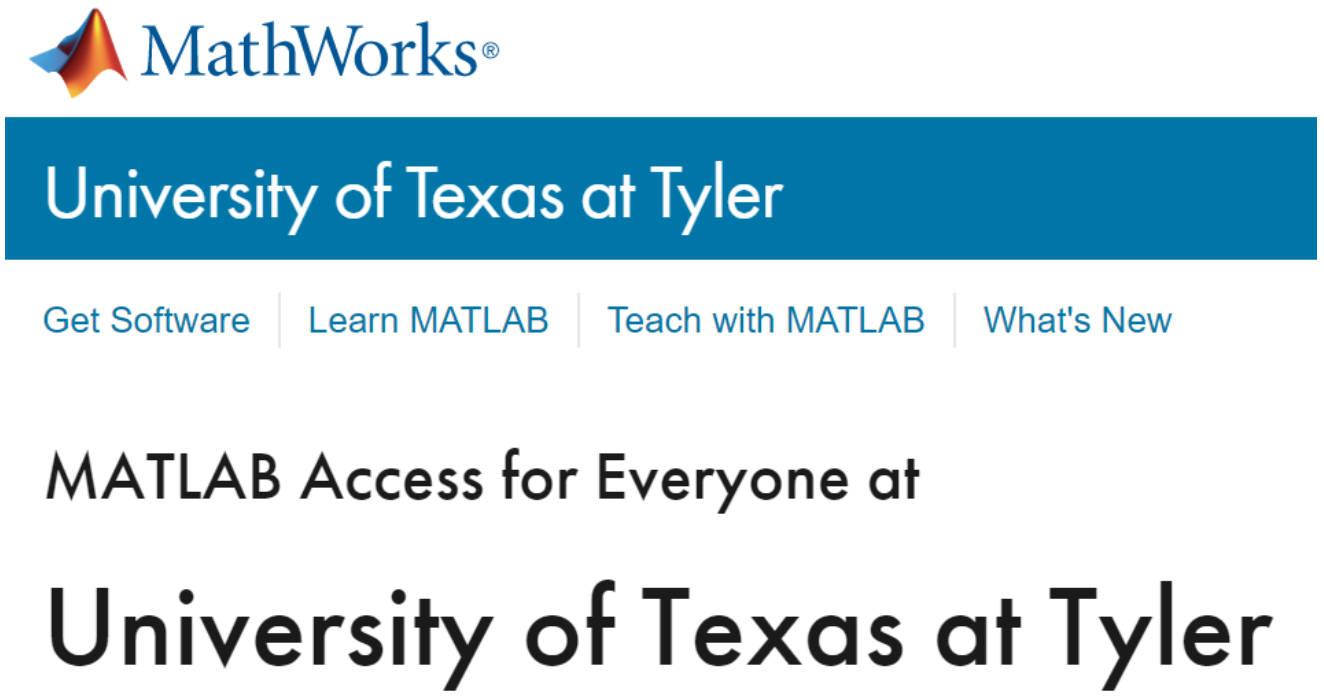
PREMANANDA INDIC, PH.D.

DEPARTMENT OF ELECTRICAL ENGINEERING

Research Design & Data Analysis Lab

Office of Research, Scholarship, and Sponsored Programs

ANALYSIS PLATFORM



https://www.mathworks.com/academia/tah-portal/university-of-texas-at-tyler-1108545.html

ANALYSIS PLATFORM

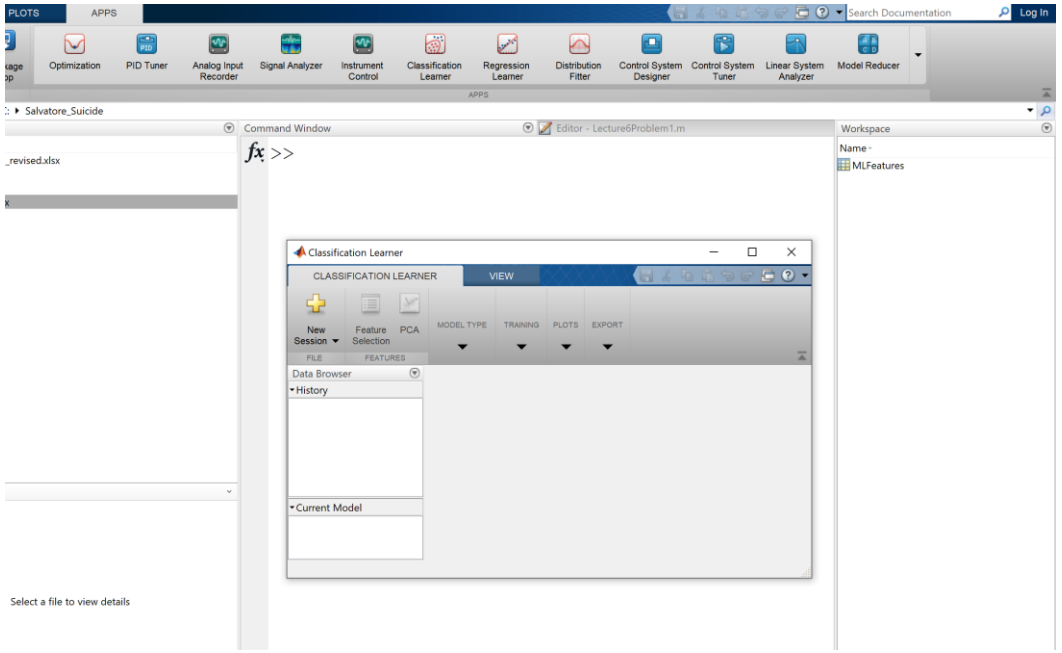
MathWorks®

University of Texas at Tyler

Get Software | Learn MATLAB | Teach with MATLAB | What's New

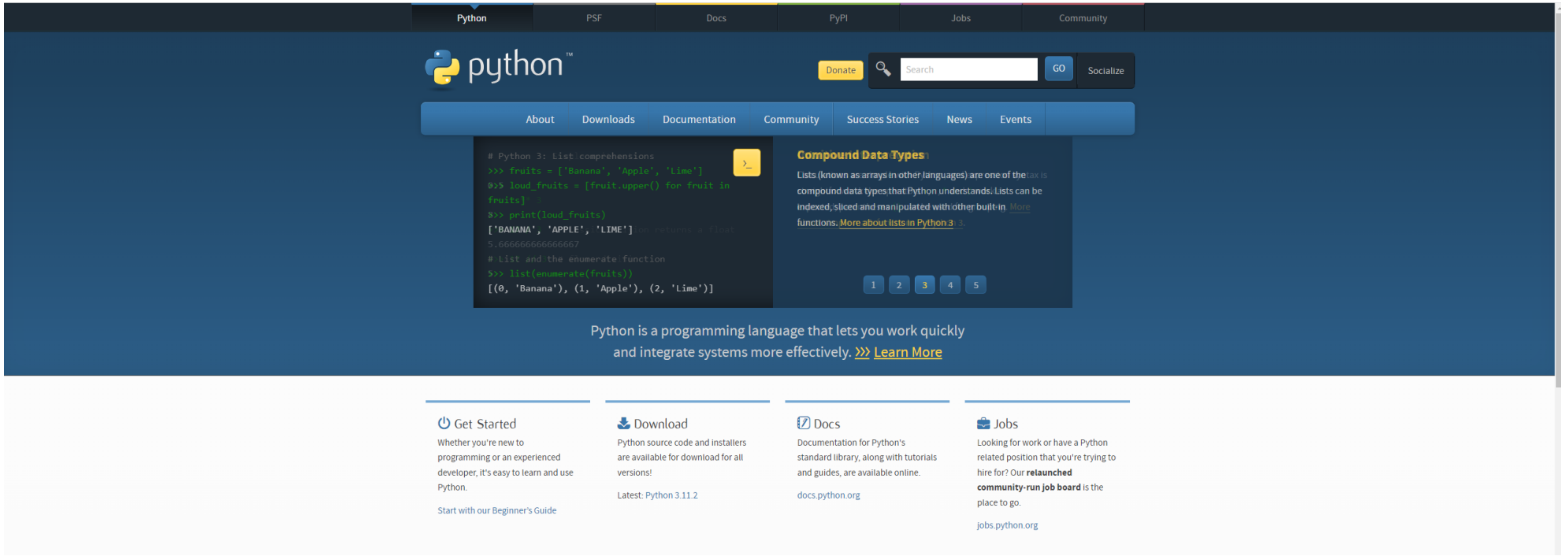
MATLAB Access for Everyone at

University of Texas at Tyler



https://www.mathworks.com/academia/tah-portal/university-of-texas-at-tyler-1108545.html

ANALYSIS PLATFORM



https://www.python.org/

OUTLINE

- INTRODUCTION
- DIFFERENT REGRESSION APPROACHES
- EXAMPLES

OUTLINE

- INTRODUCTION
- DIFFERENT REGRESSION APPROACHES
- EXAMPLES

INTRODUCTION

➢ What is Machine Learning ?

- Machine Learning is a field of study that gives computers the ability to “learn” without being explicitly programmed
  - Prediction
  - Classification

Samuel AL, IBM J. Research & Development, 1959, vol. 3 (3), 210-229

INTRODUCTION

➢What is Machine Learning ?

- Machine Learning is a field of study that gives computers the ability to “learn” without being explicitly programmed
  - Prediction (Regression)
  - Classification

Samuel AL, IBM J. Research & Development, 1959, vol. 3 (3), 210-229

OUTLINE

- INTRODUCTION
- DIFFERENT REGRESSION APPROACHES
- EXAMPLES

APPROACHES

- SUPERVISED LEARNING
- UNSUPERVISED LEARNING

APPROACHES

- SUPERVISED LEARNING (Classification / Prediction)

  Provide training set with features and solutions

APPROACHES

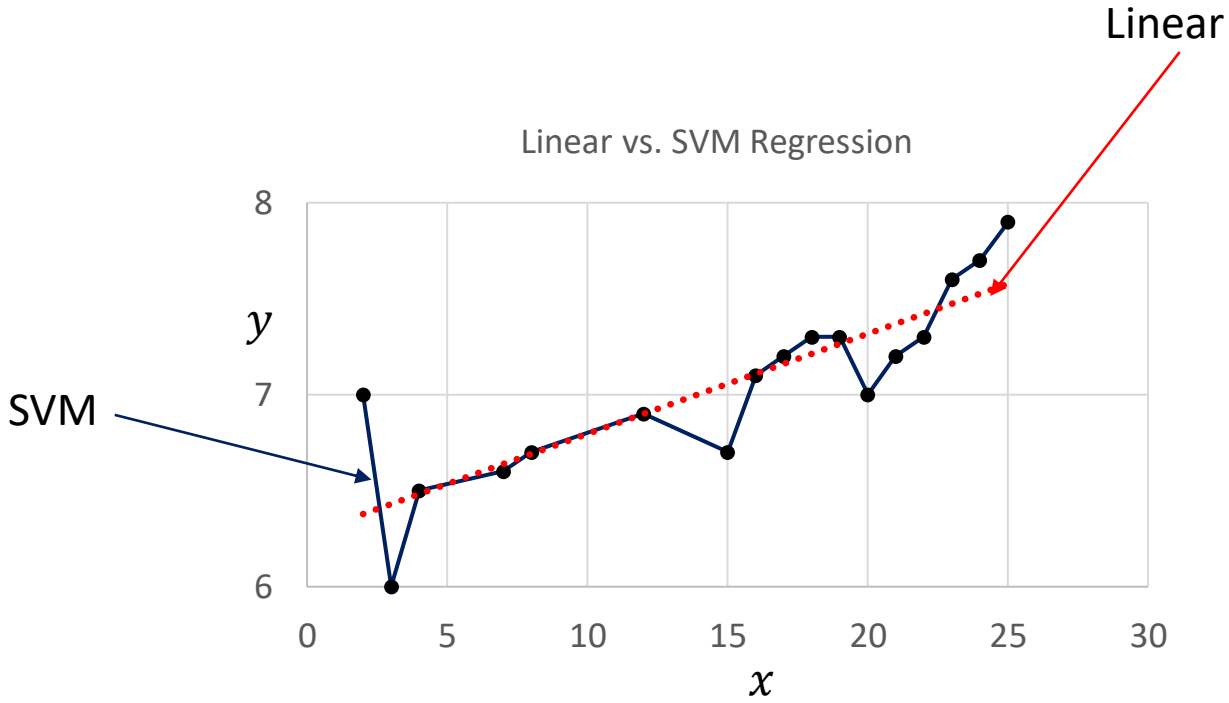
- STANDARD MACHINE LEARNING

- ADVANCED MACHINE LEARNING

  Based on Artificial Neural Networks (Deep Learning)

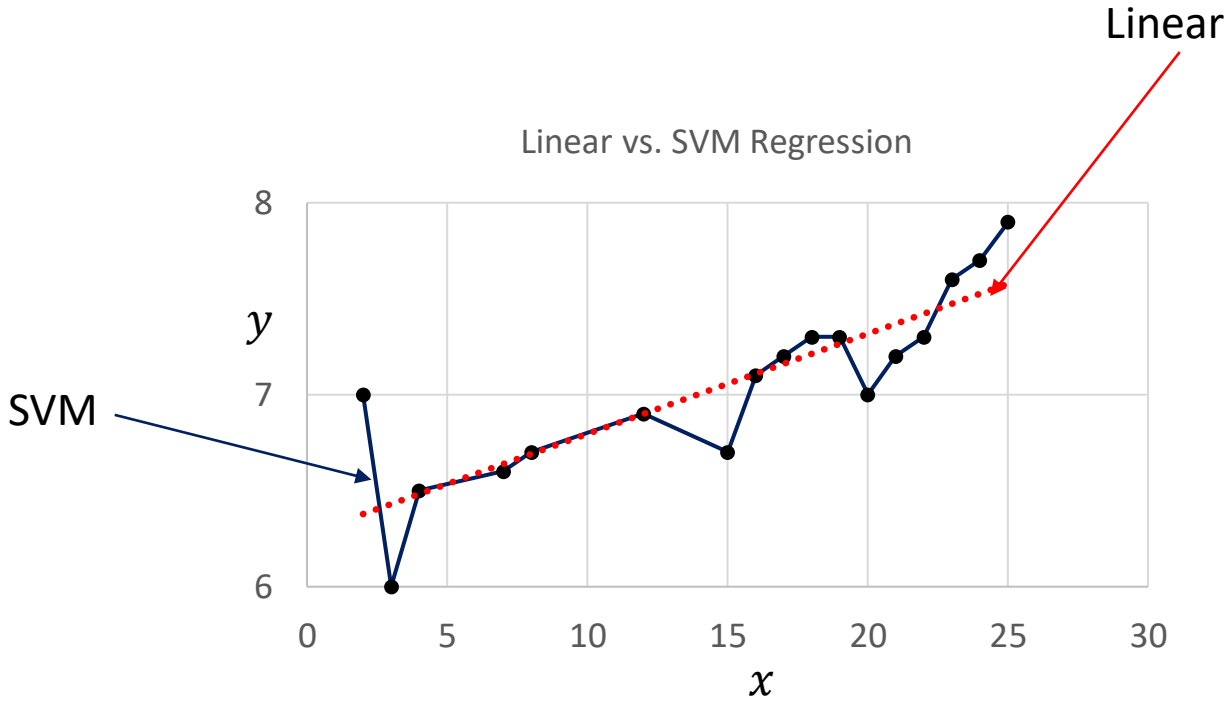
APPROACHES

- REGRESSION
  - Linear Regression
  - Support Vector Regression



APPROACHES

- REGRESSION
  - Linear Regression
  - Support Vector Regression



APPROACHES

- Linear Regression

  Given m outcomes $y^{(i)}$ where $i = 1,2, \ldots, m$ with each outcome depends on n features x_j where $j = 1,2, \ldots, n$. Find the best estimate of y^i as $\hat{y}^i$ using the n features with appropriate parameters θ_j such that $J = \left\langle \left(\hat{y}^{(i)} - y^{(i)}\right)^2 \right\rangle$

$$\hat{y}^{(i)} = \theta_0 + \theta_1 x_1^{(i)} + \theta_2 x_2^{(i)} + \cdots \ldots \ldots + \theta_n x_n^{(i)}$$

APPROACHES

- Linear Regression

$$\hat{y}^{(i)} = \theta_0 + \theta_1 x_1^{(i)} + \theta_2 x_2^{(i)} + \cdots \ldots \ldots + \theta_n x_n^{(i)}$$

$$\hat{Y} = \Theta^T X \qquad \Theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ .. \\ .. \\ \theta_n \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 1 & 1 & .. & .. & 1 \\ x_1^{(1)} & x_1^{(2)} & x_1^{(3)} & .. & .. & x_1^{(m)} \\ x_2^{(1)} & x_2^{(2)} & x_2^{(3)} & .. & .. & x_2^{(m)} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ x_n^{(1)} & x_n^{(2)} & x_n^{(3)} & .. & .. & x_n^{(m)} \end{bmatrix}$$

Cost Function to Minimize

$$J = \left\langle \left(\hat{y}^i - y^i\right)^2 \right\rangle = (\hat{Y} - Y)^T (\hat{Y} - Y)$$

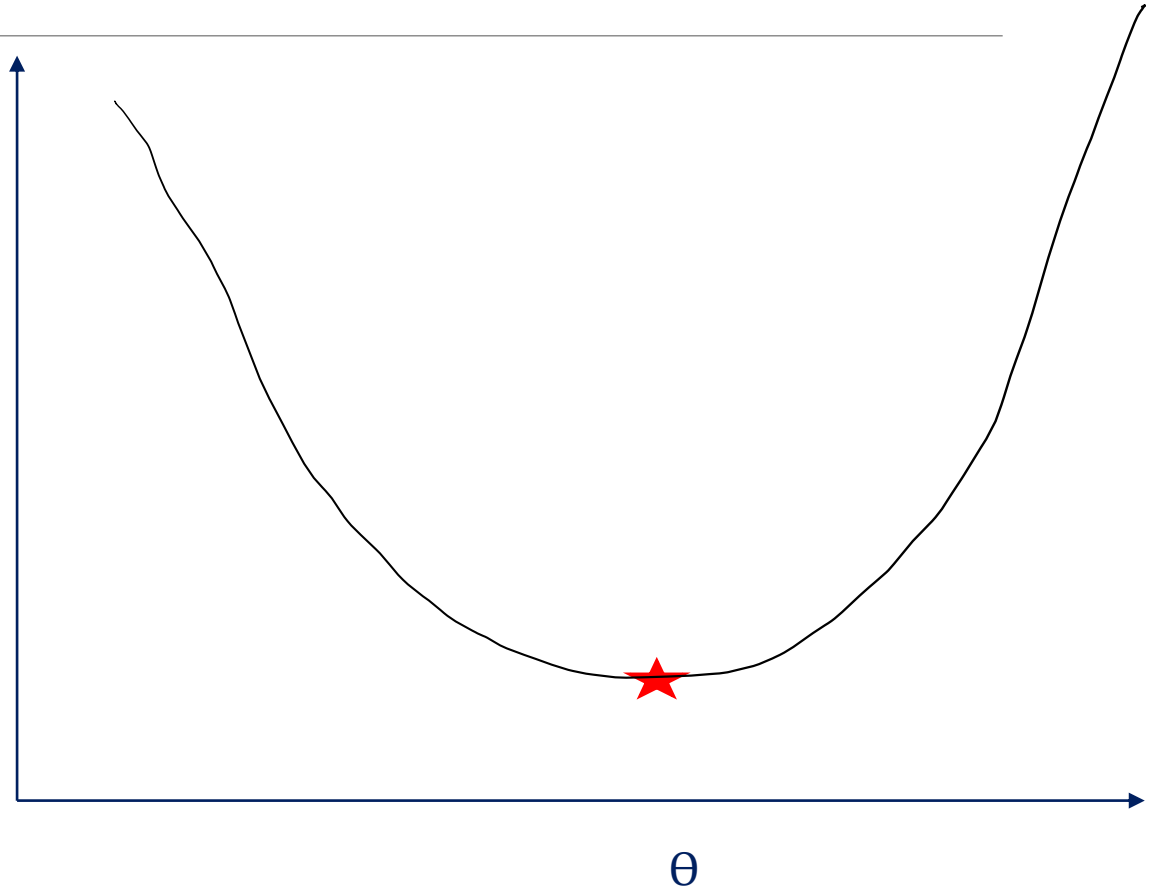
APPROACHES

- Linear Regression

$$J = \left\langle \left(\hat{y}^i - y^i\right)^2 \right\rangle = \left(\hat{Y} - Y\right)^T \left(\hat{Y} - Y\right) = (\Theta^T X - Y)^T (\Theta^T X - Y)$$

$$\frac{dJ}{d\Theta} = 0$$

$$\Theta = (X^T X)^{-1} X^T Y$$



APPROACHES

- Linear Regression

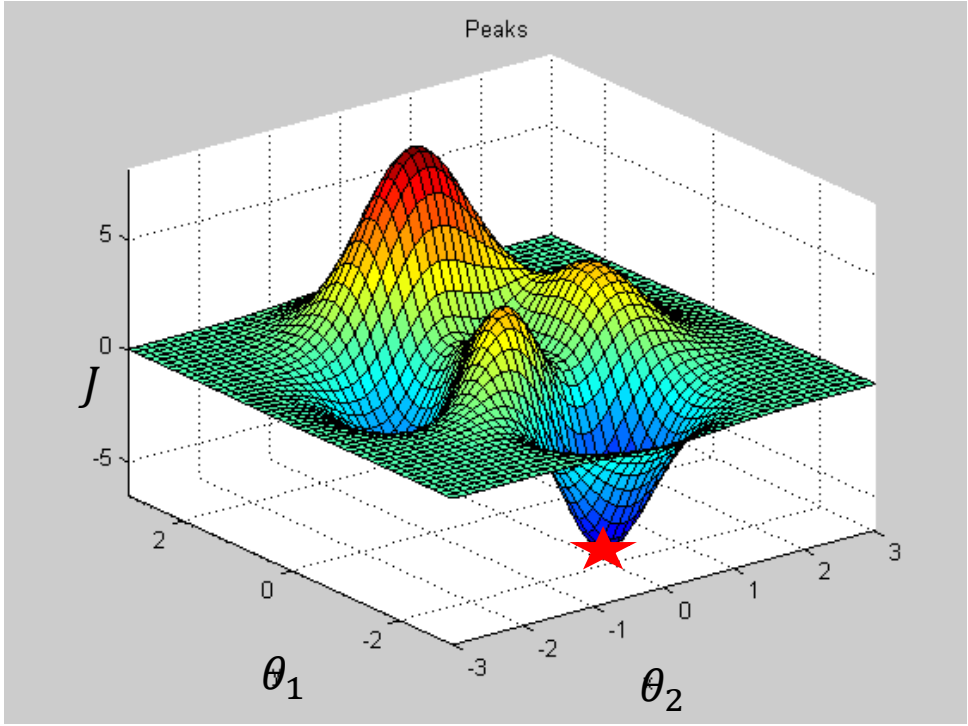
$$\hat{y}^i = \theta_0 + \theta_1 x_1^i + \theta_2 x_2^i + \cdots \ldots \ldots + \theta_n x_n^i$$

$$\hat{Y} = \Theta^T X$$

- Gradient Descent by **Louis Augustin Cauchy** in 1847

Cost Function to Minimize

$$J = \left\langle \left(\hat{y}^i - y^i\right)^2 \right\rangle = (\hat{Y} - Y)^T (\hat{Y} - Y)$$

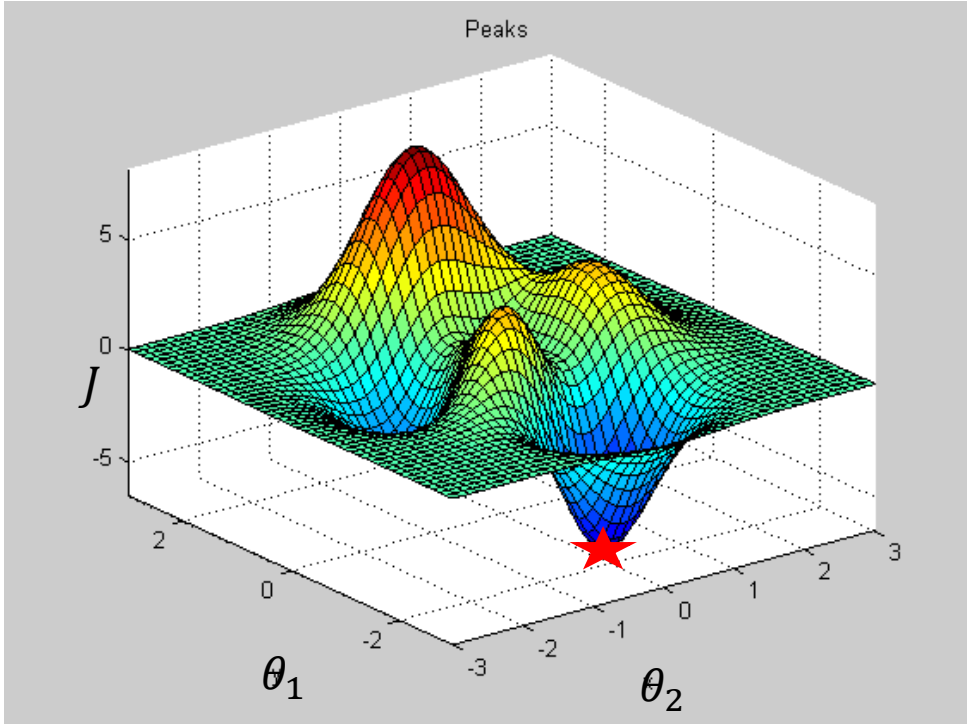


APPROACHES

- Linear Regression

$$\Theta^{k+1} = \Theta^{k} - \gamma \nabla_{\Theta} J(\Theta)$$

$$\nabla_{\Theta} J(\Theta) = \frac{2}{m} X^{T}(X\Theta - Y)$$



APPROACHES

- Polynomial Regression

Given m outcomes $y^{(i)}$ where $i = 1,2, \ldots, m$ with each outcome depends on n features x_j where $j = 1,2, \ldots, n$. Find the best estimate of y^i as $\hat{y}^i$ using the n features with appropriate parameters θ_j such that $J = \left\langle \left(\hat{y}^{(i)} - y^{(i)}\right)^2 \right\rangle$

$$\hat{y}^{(i)} = \theta_0 + \theta_1 x_1^{(i)} + \theta_2 x_1^{2(i)} + \cdots \ldots \ldots + \theta_n x_1^{n(i)}$$

APPROACHES

- Polynomial Regression

$$\hat{y}^{(i)} = \theta_0 + \theta_1 x_1^{(i)} + \theta_2 x_1^{2(i)} + \cdots \ldots \ldots + \theta_n x_1^{n(i)}$$

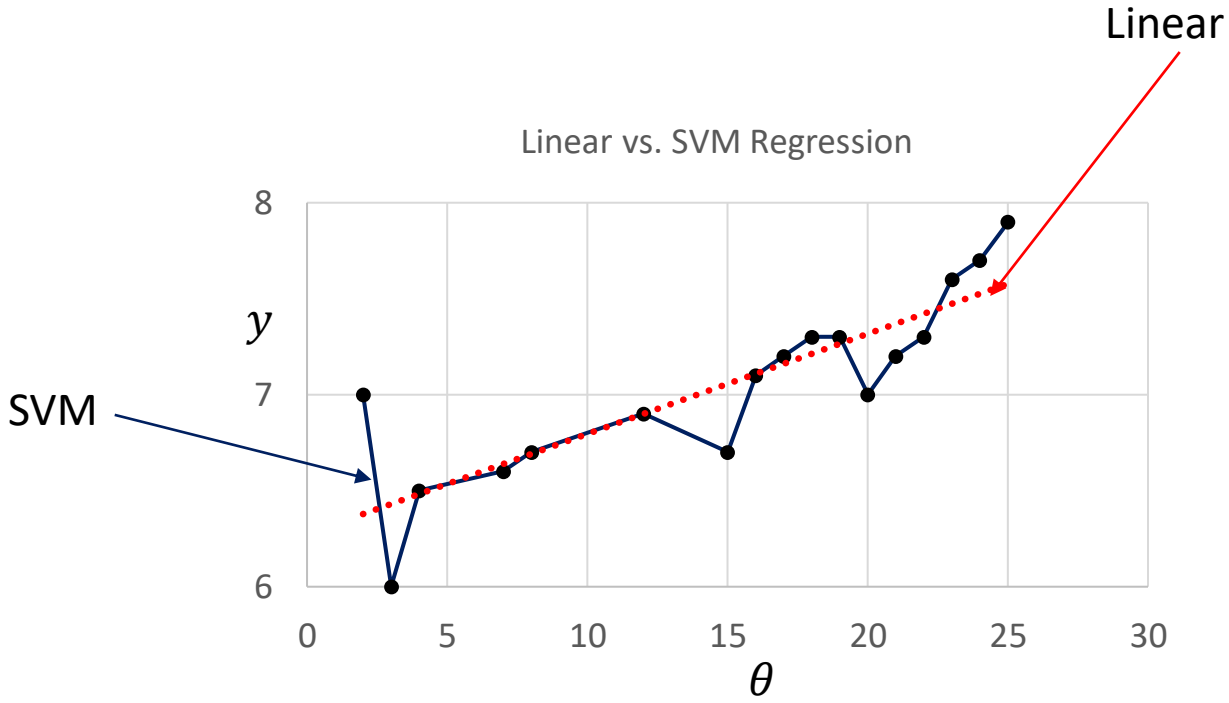
$$\hat{Y} = \Theta^T X \qquad \Theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ .. \\ .. \\ \theta_n \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 1 & 1 & .. & .. & 1 \\ x_1^{(1)} & x_1^{(2)} & x_1^{(3)} & .. & .. & x_1^{(m)} \\ x_1^{2(1)} & x_1^{2(2)} & x_1^{2(3)} & .. & .. & x_1^{2(m)} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ x_1^{n(1)} & x_1^{n(2)} & x_1^{n(3)} & .. & .. & x_1^{n(m)} \end{bmatrix}$$

Cost Function to Minimize

$$J = \left\langle \left(\hat{y}^i - y^i\right)^2 \right\rangle = (\hat{Y} - Y)^T (\hat{Y} - Y)$$

APPROACHES

- REGRESSION
  - Linear Regression
  - Support Vector Regression



APPROACHES

- Support Vector Regression

$$-\epsilon < y - f(x) < \epsilon$$

$f(x) = \theta_0 + \theta x$ (Linear Regression)

$$f(x) = \theta_0 + \sum_{i=1}^{m} G(x^i, x)$$

$G(x^i, x)$=$x^i . x$ (Linear SVR)

$G(x_j, x_k) = \exp\left(-\|x_j - x_k\|^2\right)$
$G(x_j, x_k) = (1 + x_j' x_k)^q$, where q is in the set {2,3,...}.

EXAMPLE 1

- Home Value Prediction (App Based): 9 features to predict medianHouseValue (N=20640)

longitude: A measure of how far west a house is; a higher value is farther west

latitude: A measure of how far north a house is; a higher value is farther north

housingMedianAge: Median age of a house within a block; a lower number is a newer building

totalRooms: Total number of rooms within a block

totalBedrooms: Total number of bedrooms within a block

population: Total number of people residing within a block

households: Total number of households, a group of people residing within a home unit, for a block

medianIncome: Median income for households within a block of houses (measured in tens of thousands of US Dollars)

medianHouseValue: Median house value for households within a block (measured in US Dollars)

oceanProximity: Location of the house w.r.t ocean/sea

DEMO

https://www.kaggle.com/camnugent/california-housing-prices

Demo with N=5000
70% Training Data
30% Test Data
Models Trained:
Linear Regression
SVM

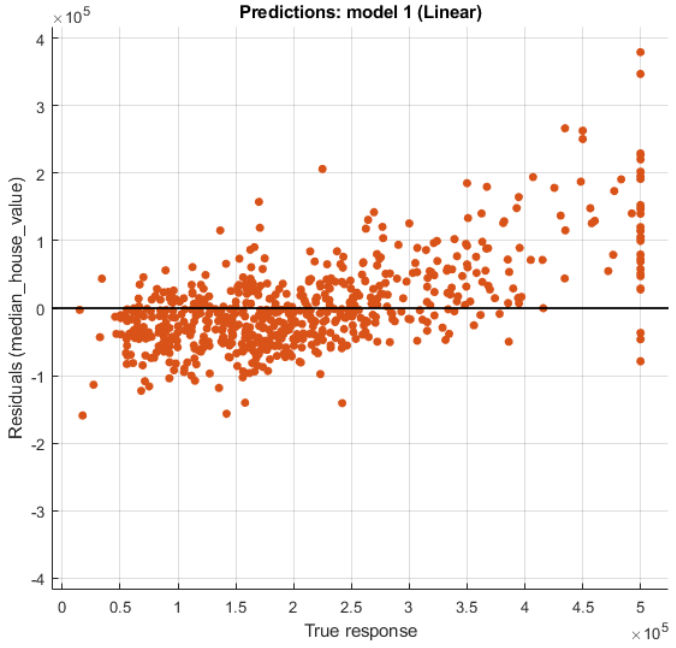
EXAMPLE 1

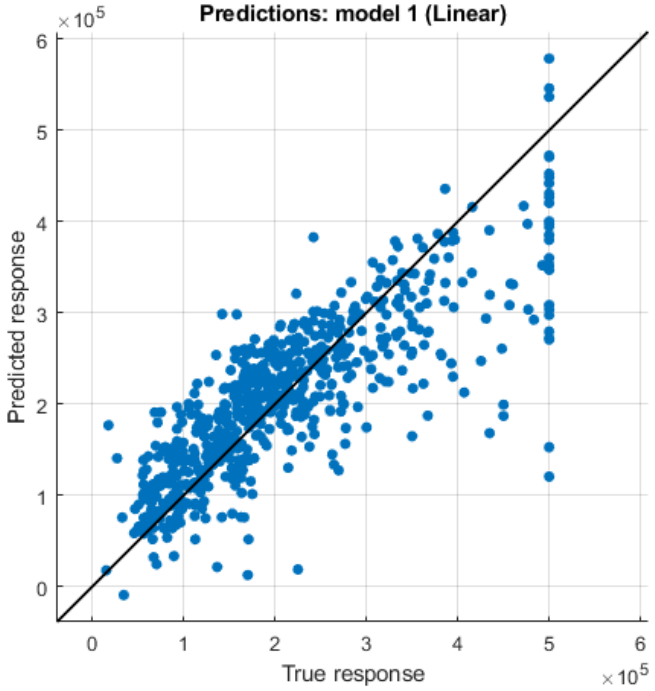
- Home Value Prediction (App Based): 9 features to predict medianHouseValue (N=5000)

Model Type	Validation (10 fold) RMSE	R-squared	Test RMSE	Test R-squared
Linear Regression (using App)	69010	0.64	65501	0.67
Linear SVM (using App)	70382	0.64	66858	0.66

EXAMPLE 1

- Home Value Prediction (App Based): 9 features to predict medianHouseValue (N=5000)





EXAMPLE 2

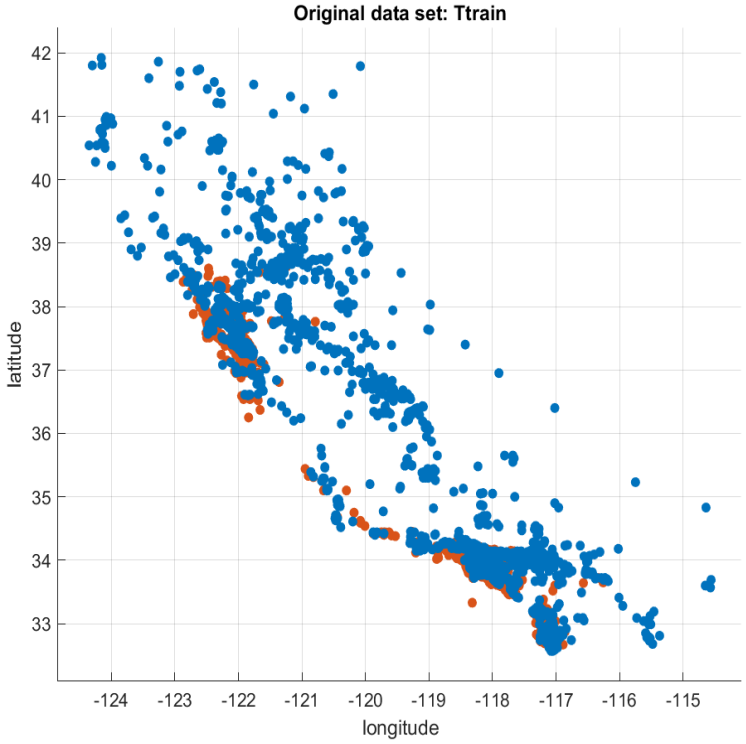
- Home Value Prediction (Realistic Approach): 9 features to predict medianHouseValue (N=5000)

1. Visualize the data
2. Identify the features (find correlations between variables)
3. Preprocess the data (missing values, outliers)
4. Train the Model
5. Select the best performance model

EXAMPLE 2

- Home Value Prediction (Realistic Approach): 9 features to predict medianHouseValue (N=5000)

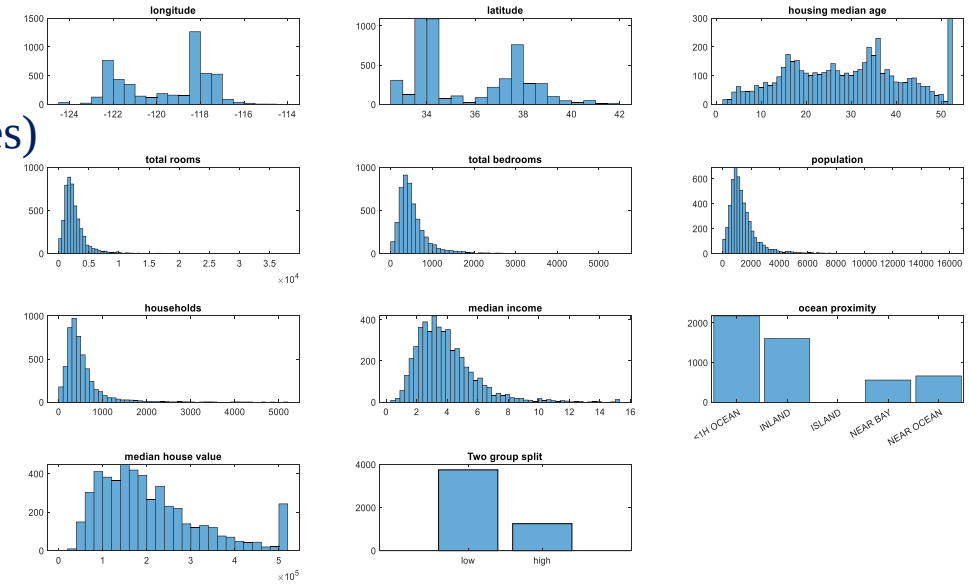
1. Visualize the data
2. Identify the features (find correlations between variables)
3. Preprocess the data (missing values, outliers)
4. Train the Model
5. Select the best performance model



EXAMPLE 2

- Home Value Prediction (Realistic Approach): 9 features to predict medianHouseValue (N=5000)

1. Visualize the data
2. Identify the features (find correlations between variables)
3. Preprocess the data (missing values, outliers)
4. Train the Model
5. Select the best performance model



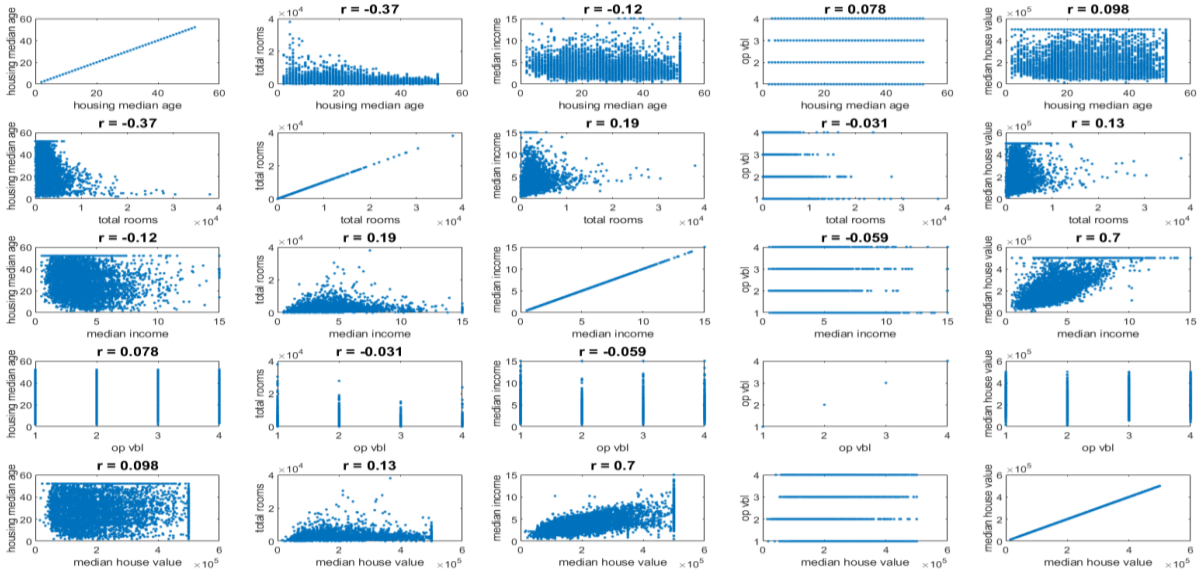
Visualize the data, Summarize variables, data cleaning, pre-processing if needed

EXAMPLE 2

- Home Value Prediction (Realistic Approach): 9 features to predict medianHouseValue (N=5000)

1. Visualize the data
2. Identify the features (find correlations between variables)
3. Preprocess the data (missing values, outliers
4. Train the Model
5. Select the best performance model

FIND VARIABLE CORRELATIONS TO EACH OTHER
AND THE MEDIAN_HOUSE_VALUE



EXAMPLE 2

- Home Value Prediction (Realistic Approach): 9 features to predict medianHouseValue (N=5000)

1. Visualize the data
2. Identify the features (find correlations between variables)
3. Preprocess the data (missing values, outliers)
4. Train the Model
5. Select the best performance model

207 Missing values, replace with median values

ocean_proximity: 20636×1 categorical

Values:

<1H OCEAN	9135
INLAND	6550
ISLAND	5
NEAR BAY	2289
NEAR OCEAN	2657

```
Visualize the data, Summarize variables, data cleaning, pre-processing if needed
```

EXAMPLE 2

- Home Value Prediction (Realistic Approach): 9 features to predict medianHouseValue (N=5000)

1. Visualize the data
2. Identify the features (find correlations between variables)
3. Preprocess the data (missing values, outliers)
4. Train the Model **DEMO**
5. Select the best performance model

Linear Regression Fewer Variables RMSE ~69100

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-2.3266e+06	2.011e+05	-11.57	2.0947e-30
longitude	-27661	2340.9	-11.816	1.2823e-31
latitude	-26535	2321.7	-11.43	9.9957e-30
housing_median_age	1014	104.58	9.6958	5.9307e-22
total_rooms	-3.6077	1.7753	-2.0322	0.042206
total_bedrooms	101.37	16.167	6.2701	4.0505e-10
population	-42.973	2.7491	-15.632	2.7235e-53
households	44.258	18.03	2.4547	0.014149
median_income	38847	799.97	48.56	0
op_inland	-38746	4137.6	-9.3641	1.3342e-20

Number of observations: 3500, Error degrees of freedom: 3490
Root Mean Squared Error: 6.91e+04
R-squared: 0.645, Adjusted R-Squared 0.644
F-statistic vs. constant model: 704, p-value = 0

SPLIT INTO TRAINING AND TEST DATA AND FIT REGRESSION MODELS

EXAMPLE 2

- Home Value Prediction (Realistic Approach): 9 features to predict medianHouseValue (N=5000)

1. Visualize the data
2. Identify the features (find correlations between variables)
3. Preprocess the data (missing values, outliers)
4. Train the Model
5. Select the best performance model

EXAMPLE 2

- Home Value Prediction (Realistic Approach): 9 features to predict medianHouseValue (N=5000)

Model Type	Validation RMSE	Test RMSE
Lin regression	70071	65501
Lin. Regression – fewer variables	69031	65357
SVM –linear kernel	116370	116130
SVM –Gaussian Kernel	60099	57708

LASSO REGRESSION

- Linear Regression

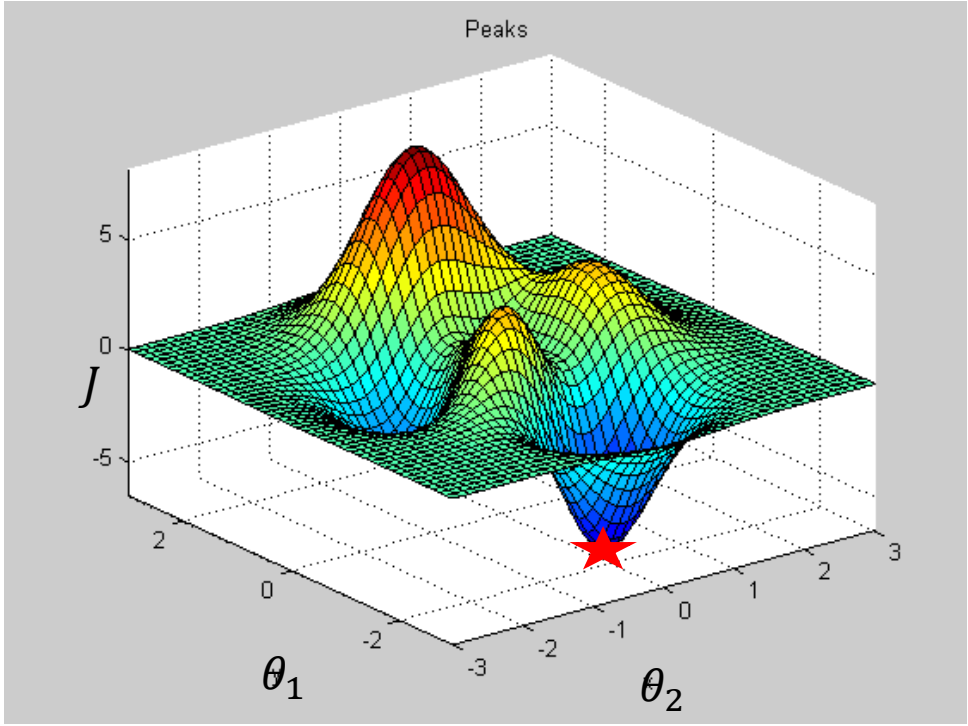
$$\hat{y}^i = \theta_0 + \theta_1 x_1^i + \theta_2 x_2^i + \cdots \ldots \ldots + \theta_n x_n^i$$

$$\hat{Y} = \Theta^T X$$

- Gradient Descent by **Louis Augustin Cauchy** in 1847

Cost Function to Minimize

$$J = \left\langle \left(\hat{y}^i - y^i\right)^2 \right\rangle = (\hat{Y} - Y)^T (\hat{Y} - Y)$$

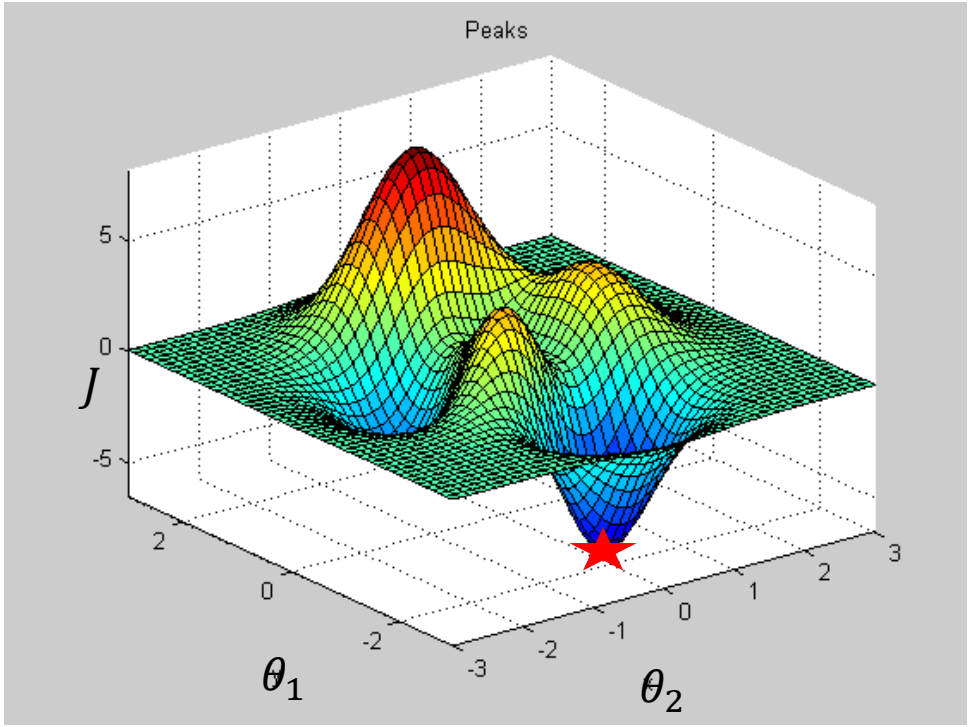


LASSO REGRESSION

- Linear Regression with Lasso

$$\hat{y}^i = \theta_0 + \theta_1 x_1^i + \theta_2 x_2^i + \cdots \ldots \ldots + \theta_n x_n^i$$

$$\hat{Y} = \Theta^T X$$

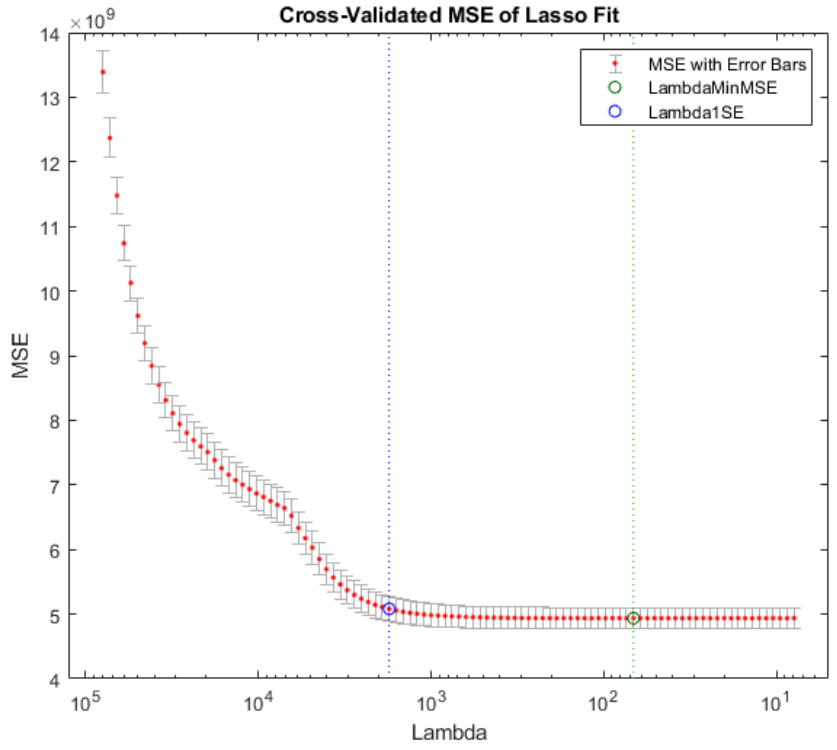


Cost Function to Minimize

$$J = \left\langle \left(\hat{y}^i - y^i\right)^2 \right\rangle = \left(\hat{Y} - Y\right)^T \left(\hat{Y} - Y\right) + \lambda \sum_{j=1}^{n} |\theta_j|$$

EXAMPLE 3

- Home Value Prediction (Lasso Regression): 9 features to predict medianHouseValue (N=5000)



$$J = \left\langle \left(\hat{y}^i - y^i\right)^2 \right\rangle = \left(\hat{Y} - Y\right)^T \left(\hat{Y} - Y\right) + \lambda \sum_{j=1}^{n} |\theta_j|$$

Lambda

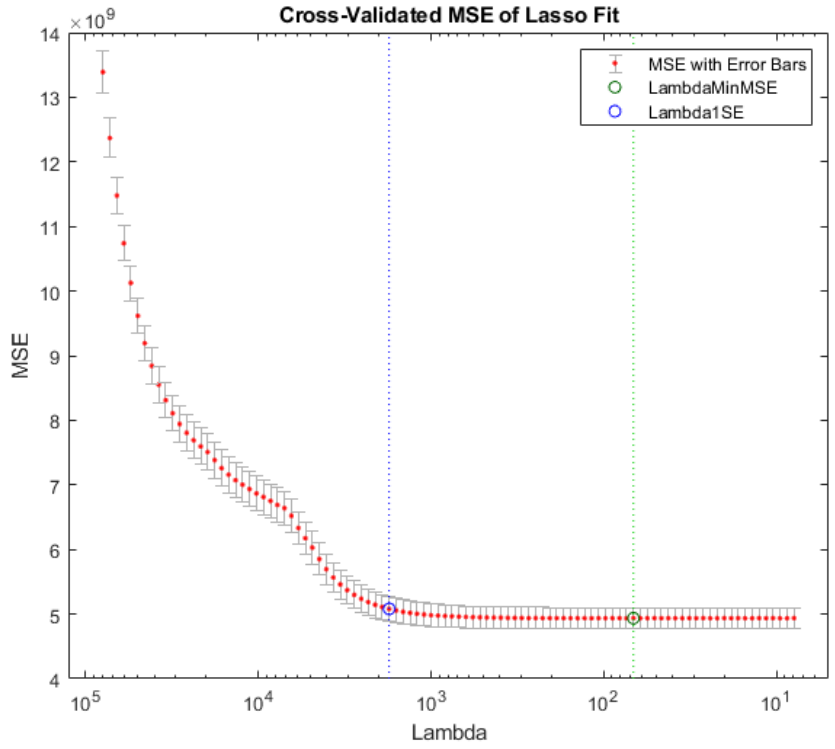
Lasso removes the 'total_rooms' and 'Ocean Proximity_inland' variables as least important.

RMSE on test data with 7 features = 66443

DEMO

EXAMPLE 3

- Home Value Prediction (Lasso Regression): 9 features to predict medianHouseValue (N=5000)



{'longitude’	-3.2643
'latitude’	-3.2856
'housing_median_age’	0.1177
'total_rooms’	0
'total_bedrooms’	0.0074
'population’	-0.0028
'households’	0.0014
'median_income’	3.8702
'op_vbl'}	0

All coefficients multiplied by 10.^4

CONCLUSION

- Regression provides continuous prediction of an outcome with selected features
- Understanding of features in relation to outcome is important
- Several codes are available to perform regression analysis

UTTyler Center for Health Informatics & Analytics

THANK YOU

SBIR: RAE (Realize, Analyze, Engage) - A digital biomarker based detection and intervention system for stress and carvings during recovery from substance abuse disorders.
PIs: M. Reinhardt, S. Carreiro, P. Indic

STARs Award

The University of Texas System
P. Indic (PI, UT Tyler)

Research Design & Data Analysis Lab
Office of Research, Scholarship, and Sponsored Programs

Department of Veterans Affairs

Design of a wearable sensor system and associated algorithm to track suicidal ideation from movement variability and develop a novel objective marker of suicidal ideation and behavior risk in veterans.
Clinical Science Research and Development Grant (approved for funding),
P. Indic (site PI, UT-Tyler)
E.G. Smith (Project PI, VA)
P. Salvatore (Investigator, Harvard University)

Pre-Vent

National Institute Of Health Grant
P. Indic (Analytical Core PI, UT-Tyler)
N. Ambal (PI, Univ. of Alabama, Birmingham)

ViSiON

National Institute Of Health Grant
P. Indic (Co-Investigator & site PI , UT-Tyler)
P. Ramanand (Co-Investigator, UT-Tyler)
N. Ambal (PI, Univ. of Alabama, Birmingham)

QUESTIONS